

# Water resources valuation in competitive environments

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## Abstract

This paper analyses the assessment of water resources in competitive generation power markets. With that purpose, the centralized definition of water value is updated and extended in order to adapt it to electricity markets, where generation companies face the hydro-thermal coordination. Two different water values are defined as well as their applications: profit value and cost value. A methodology to compute both water values is presented, and its application to a case study.

**Keywords:** Electricity markets, Hydro-thermal coordination, Market equilibrium, Water value.

## 1. Introduction

During the last years, important reorganizations have been taking place in the electricity sector of many countries. There is a pronounced trend towards liberalization and decentralization. Liberalized electrical markets assign major roles to generation companies that face a number of strategic decisions with a different time scale.

One of the most important decisions that the company makes to operate its units is the hydro-thermal coordination. The decisions regarding hydraulic resources allocation are taken in a medium-term scope (one year in many systems, even though there exist hydro systems with a longer time scale). Subsequently, the programming of the units is made in a short-term scope (on a weekly basis usually). Therefore, the medium-term planning must provide the short-term operation with the right signals, in order to satisfactorily achieve hydraulic coordination.

In most cases, two different signals are used from medium-term planning toward short-term operation in centralized environments, where the single market operator tries to minimize total system costs:

- The total quantity of available energy to be used during one week (or the adequate short-term time scale).

- The water value: the substitution water cost (total cost variation caused by hydraulic resources variation) [1]. It is important to distinguish between cost (which is almost zero) and value. Another important issue when mentioning water value, is that we are referring to marginal water value.

If the forecasted conditions in the medium-term planning correspond to the actual conditions that occur in the short-time operation, both signals are equivalent. However this never occurs, for two main reasons. The first one is that the medium-term model is a simplification of the reality, so it eliminates details which became important in the short term (thermal groups ramps, low-level topology of the hydraulic subsystems, etc.); however, these simplifications allow achieving an adequate planning in the medium term. The second reason is that medium-term forecasts will not correspond to the actual conditions. The main sources of uncertainty are commonly the demand, hydro inflows, the availability of generation units, and fuel costs.

The short-term programming is carried out facing different conditions than those considered in the medium-term planning. If the difference between both conditions is low, acceptable results could be obtained imposing the available hydro energy for the week. In any case, better results will be obtained in the short-term programming by using an explicit valuation of the hydraulic resources. This valuation is brought on by the water value, which internalizes the hydro-thermal coordination during the medium-term horizon, as well as the representation of the hydraulic subsystems characteristics.

There are few papers that have addressed water resources valuation in competitive environments. The first work including water value in electricity markets is [2]. The author applies Dual Dynamic Programming, using water value to compute market equilibrium. Nevertheless, special attention it is not paid to the meaning and significance of water value. Water value is defined in an explicit way in [3], though the applications are limited because does not allow the consideration of real-size systems. However, this is the first approach that clearly defines the water value and its significance in

competitive environments. In [4], market equilibrium is computed using Mixed Complementarity Problem (MCP), allowing for the computation of water value. Finally, market equilibrium is stated through Dynamic Programming in [5], with the possibility of computing water value as the derivative of future-cost function.

Nevertheless, all these works do not clearly state the meaning and the significance of water value in competitive environments. This paper extends the traditional definition of water value, in order to adapt it to electricity markets, where generation companies face the hydro-thermal coordination. Two different water values are defined, as well as their applications: profit value and cost value. A methodology to compute both water values is presented, as well as its application to a case study

## 2. Water value definition in competitive environments

### A. Water value in centralized environments

In a traditional centralized framework, the system operator's objective is to minimize the total generation costs. Let  $c=1\dots C$  be the companies and  $p=1\dots P$  the time periods with a length of  $l_p$  hours. Considering power-balance constraint and a unique hydro unit, the mathematical formulation for minimizing total costs is as follows:

$$\begin{aligned} \min_{T_{cp}, H_p, R_p} \quad & \sum_{c=1}^C \sum_{p=1}^P C_{cp}(T_{cp}) \\ \text{s.t.} \quad & \sum_{c=1}^C T_{cp} + H_p = D_p \quad : \quad \eta_p \\ & R_p - R_{p-1} = I_p - H_p \cdot l_p \quad : \quad \mu_p \\ & R_p = R_f \\ & \underline{R}_p \leq R_p \leq \bar{R}_p \end{aligned} \quad (1)$$

where  $T_{cp}$  is the thermal generation for company  $c$  at period  $p$ ;  $C_{cp}$  is the thermal generation cost for company  $c$  at period  $p$ ;  $H_p$  is the hydro generation at period  $p$ ;  $D_p$  is the total system demand (considered as inelastic) at period  $p$ ;  $R_p$  is the reservoir level for the hydro unit at the end of period  $p$ ;  $I_p$  is the hydro inflows at period  $p$ ;  $R_f$  is the final reservoir level for the hydro unit; and the reservoir limits for the hydro unit are  $\underline{R}_p$  and  $\bar{R}_p$ .

The dual variable  $\eta_p$  of the power-balance constraint is, by definition, the derivative of the total cost with respect to the system demand. This is to say, the marginal cost  $\beta_p$  for every time period can be obtained as:

$$\beta_p = \frac{\eta_p}{l_p} \quad (2)$$

In addition, the dual variable  $\mu_p$  of the hydro balance constraint is the derivative of the total cost with respect to the hydro inflows at period  $p$ . Its absolute value ( $\mu_p$  is non positive) has been traditionally called **water value**. In other words, the water value is defined as the

substitution cost of the thermal units replaced by hydro generation. It constitutes a very useful tool in medium- and short-term operation of power systems.

### B. Water value in competitive environments

The above definition of water value loses its meaning in competitive market environments, considering that there is no longer a centralized system operator which makes up the generation planning. In this new context, each company manages its hydraulic resources in order to maximize its respective benefits. That is the reason why water value definitions have to be updated according to these new aspects.

This paper exposes two water value definitions:

- **Profit water value** is defined as the variation of the company's profit with respect to its available hydraulic resources. That is to say, the unitary change in the company's profit when varying its hydro resources.
- **Cost water value** is defined as the substitution value for the company that owns the hydro resource. This is to say, the cost avoided by the company when hydro generation is unitarily incremented. Similarly to the considerations in a centralized environment, a company will use hydro generation from a marginal cost that corresponds to the cost water value. In this paper, it is demonstrated that, in the short term, companies cope with a cost minimization facing a residual demand. Hence, the situation can be considered equivalent to a centralized environment. Thus, cost water value is the correct economic signal that allows for coordination among medium-term and short-term hydro operation. A method to compute this water value is presented.

## 3. Computation of water value

### A. Market equilibrium model

This paper makes use of a representation of electricity markets as a market-equilibrium problem, solved through an equivalent optimization problem based on a conjectural-variation approach, as stated in [6], [7]. It has been proved that, under some reasonable assumptions, the market equilibrium can be computed by solving an equivalent quadratic optimization problem:

$$\begin{aligned} \min_{T_{cp}, H_{cp}, R_p} \quad & \sum_{c=1}^C \sum_{p=1}^P \bar{C}_{cp}(T_{cp}, H_{cp}) \\ \text{s.t.} \quad & \sum_{c=1}^C (T_{cp} + H_{cp}) = D_p \quad : \quad \eta_p \\ & R_{cp} - R_{c,p-1} = I_{cp} - H_{cp} \cdot l_p \quad : \quad \mu_{cp} \\ & \text{Hydro Constraints} \end{aligned} \quad (3)$$

where  $T_{cp}$  and  $H_{cp}$  represent the thermal and hydro production of company  $c$  at period  $p$ .  $\bar{C}_{cp}(\bullet)$  denotes the so-called *effective cost function*:

$$\bar{C}_{cp}(T_{cp}, H_{cp}) = C_{cp}(T_{cp}) + \frac{(T_{cp} + H_{cp})^2 \cdot \theta_{cp}}{2} \quad (4)$$

being  $\theta_{cp}$  the conjectured variation of the clearing price  $\lambda_p$  with respect to each firm production.

$$\theta_{cp} = -\frac{\partial \lambda_p}{\partial (T_{cp} + H_{cp})} \quad (5)$$

These conjectured variations are assumed to be known. The system marginal price can be computed from the dual variable of the power-balance constrain:

$$\lambda_p = \frac{\eta_p}{l_p} \quad (6)$$

### B. Computation and significance of cost water value

The proposed medium-term model allows directly obtaining cost water value from computing market equilibrium. Hence, as shown in [7], [8], the cost water value  $\delta$  is the absolute value of the dual variable of the hydro-balance constraints:

$$\delta_{cp} = \frac{\partial C_c}{\partial I_{cp}} = -\mu_{cp} \quad (7)$$

This result is an extension of the one obtained in the centralized case.

Next, the meaning of the cost water value will be treated. Considering that a) only one company ( $c=1$ ) owns hydro production, b) there are no reservoir limits, and c)  $l_p=1$  for  $p=1\dots P$ , then market equilibrium is computed though the following minimization problem:

$$\begin{aligned} \min_{T_{cp}, H_{1p}} \quad & \sum_{p=1}^P \left[ C_{1p}(T_{1p}) + \frac{\theta_{1p} \cdot (T_{1p} + H_{1p})^2}{2} \right] \\ & + \sum_{p=1}^P \sum_{c=2}^C \left[ C_{cp}(T_{cp}) + \frac{\theta_{cp} \cdot T_{cp}^2}{2} \right] \\ \text{s.t.} \quad & \sum_{c=1}^C (T_{cp}) + H_{1p} = D_p \quad : \quad \lambda_p \\ & \sum_{p=1}^P H_{1p} = I \quad : \quad \mu \end{aligned} \quad (8)$$

where hydro constraints has been taken into account and  $I$  are the total inflows in the medium-term scope.

The solution of an optimization problem does not change if some variables are substituted for their value in the optimum<sup>1</sup>. Denoting  $T_{cp}^*$  as optimal thermal productions, the market equilibrium can be computed as:

$$\begin{aligned} \min_{T_{1p}, H_{1p}} \quad & \sum_{p=1}^P \left[ C_{1p}(T_{1p}) + \frac{\theta_{1p} \cdot (T_{1p} + H_{1p})^2}{2} \right] \\ \text{s.t.} \quad & T_{1p} + H_{1p} = D_p - \sum_{c=2}^C (T_{cp}^*) \quad : \quad \lambda_p \quad (9) \\ & \sum_{p=1}^P H_{1p} = I \quad : \quad \mu \end{aligned}$$

where constants have been eliminated from the objective function.

Let now  $D_p^*$  be the residual demand for  $c=1$ ; this is to say, the system demand minus the rest of the companies' production. Then, the quadratic term can be eliminated from the objective function, so the problem is equivalent to:

$$\begin{aligned} \min_{T_{1p}, H_{1p}} \quad & \sum_{p=1}^P [C_{1p}(T_{1p})] \\ \text{s.t.} \quad & T_{1p} + H_{1p} = D_p^* \quad : \quad \lambda_p \quad (10) \\ & \sum_{p=1}^P H_{1p} = I \quad : \quad \mu \end{aligned}$$

This development shows how every company makes a cost minimization to reach its optimal production; this is to say, the market equilibrium production.

In order to clarify the meaning of cost water value, the problem is reduced to the first time period, fixing the rest of the variables to their values at the optimum.

$$\begin{aligned} \min_{T_{11}, H_{11}} \quad & C_{11}(T_{11}) \\ \text{s.t.} \quad & T_{11} + H_{11} = D_1^* \quad : \quad \lambda_1 \quad (11) \\ & H_{11} = I - \sum_{p=2}^P H_{1p} \quad : \quad \mu \end{aligned}$$

The second constraint can be incorporated in the objective function using lagrangian function:

$$\begin{aligned} \min_{T_{11}, H_{11}} \quad & C_{11}(T_{11}) - \mu \cdot H_{11} \\ \text{s.t.} \quad & T_{11} + H_{11} = D_1^* \quad : \quad \lambda_1 \quad (12) \end{aligned}$$

This equivalent problem shows that cost water value is the right valuation of the hydro resources when facing short-term operation. Therefore, cost water value constitutes the correct economic signal for coordination between medium-term hydro scheduling and short-term hydro operation [9].

An additional interpretation of the obtained results is that cost water value is the avoided thermal cost when hydro inflows are incremented. In other words, thermal units with a cost over the cost water value are replaced for hydro generation. This is the reason why it can also be called **replacement water value**.

### C. Computation and significance of profit water value

Profit water value  $\pi$  has been defined as the derivative of the company's profit  $B_c$  with respect to the available

<sup>1</sup> Theoretically, it can be guaranteed that the primal solution is the same, but not the dual. Nevertheless, in this case both solutions are the same, because the variables that "react" when computing dual variables are not the variables which have been fixed.

hydraulic resources.

$$\frac{\partial B_c}{\partial I_{cp}} = \pi_{cp} \quad (13)$$

This value will not be, in general, equal to the cost water value. Let  $x$  be a company with hydro production. Then, cost and profit water value are equal when the company  $x$  reacts to a small change in hydro inflows (this is to say, when company  $x$  does not modify its total production if hydro inflows are modified). Nevertheless, when other company reacts to the variation in hydro inflows (this is to say, when company  $x$  changes its production if hydro inflows are modified), profit value is not equal to cost value. The case study shows examples of these two situations.

Profit water value cannot be directly obtained from the computation of market equilibrium. However, it can be computed obtaining a new market equilibrium with a small variation in water inflows, from the difference in company's profit:

$$\pi_{cp} \approx \frac{\Delta B_c}{\Delta I_{cp}} \quad (14)$$

The profit water value can be useful when assessing investments in new or existing hydro installations.

## 4. Case study

### A. Case description

The case study represents a system with two generation companies. GenCo  $x$  owns 10 thermal units:  $g1$ - $g10$  and a hydro unit  $h1$ , while GenCo  $y$  owns 8 thermal units:  $g11$ - $g18$ .

TABLE I  
INSTALLED GENERATION CAPACITY (MW) AND  
GENERATION COSTS (€/MWh)

GenCo x	g1	g2	g3	g4	g5
Installed capacity	1000	1000	300	300	200
Generation cost	5	7	12	15	18
GenCo x	g6	g7	g8	g9	g10
Installed capacity	800	800	500	500	600
Generation cost	22	25	41	46	48
GenCo y	g11	g12	g13	g14	g15
Installed capacity	900	1000	300	300	800
Generation cost	4	6	16	18	22
GenCo y	g16	g17	g18		
Installed capacity	600	600	600		
Generation cost	38	41	43		

TABLE II  
CHARACTERISTICS OF HYDRO UNIT

Installed capacity (MW)	Initial and final reservoir levels (GWh)	Total inflows (GWh)
500	10	30

Table I shows the installed generation capacity and the production cost for each thermal unit. Table II shows the

main characteristics of the hydro unit (reservoir limits are shown in Figure 1, with the actual reservoir levels). The time horizon is considered to be 12 time periods  $p=p1...p12$ . Each period is supposed to be split into 5 load levels ( $l=11...15$ ). Table III shows the duration of the load levels in every period. Finally, the inelastic demand for each time period and load level, and the conjectured variation of the price with respect to each firm production are shown in Tables IV and V, respectively.

TABLE III  
LOAD LEVEL DURATION (HOURS)

l1	l2	l3	l4	l5
14	28	56	42	28

TABLE IV  
INELASTIC DEMAND (MW)

	l1	l2	l3	l4	l5
p1	8900	8200	7500	6500	5800
p2	9300	8400	7700	6800	6000
p3	9600	8700	8100	7000	6200
p4	10500	9200	8400	7200	6400
p5	10000	9000	8000	6900	6200
p6	9200	8400	7800	6700	6100
p7	8600	7800	7200	6200	5600
p8	8200	7500	6900	6000	5300
p9	8100	7300	6900	5900	5100
p10	8500	7800	7300	6200	5400
p11	8800	8000	7500	6300	5500
p12	9200	8300	7600	6600	5500

TABLE V  
CONJECTURED VARIATION OF THE PRICE WITH RESPECT  
TO THE FIRM'S PRODUCTION ((€/MWh)/GW)

	l1	l2	l3	l4	l5
GenCo x	5	5	6	8	10
GenCo y	6	4	8	6	6

### B. Results

Table VI shows the costs water value and the profit water value obtained in every period.

TABLE VI  
COST WATER VALUE AND PROFIT WATER VALUE (€/MWh)

	Cost water value	Profit water value
p1	41.00	41.20
p2	41.80	36.70
p3	45.90	41.00
p4	46.00	47.30
p5	46.00	46.00
p6	41.30	36.20
p7	41.00	41.10
p8	39.10	41.10
p9	39.10	34.00
p10	41.00	51.70
p11	41.00	0.40
p12	41.00	-9.20

Note that the cost water value remains constant in some periods (e.g.  $p_{10-p_{12}}$ ). This is because of the actual reservoir levels of the hydro unit are within their limits; therefore it is possible to transfer hydro production among them. Reservoir limits and actual reservoir levels are shown in Figure 1.

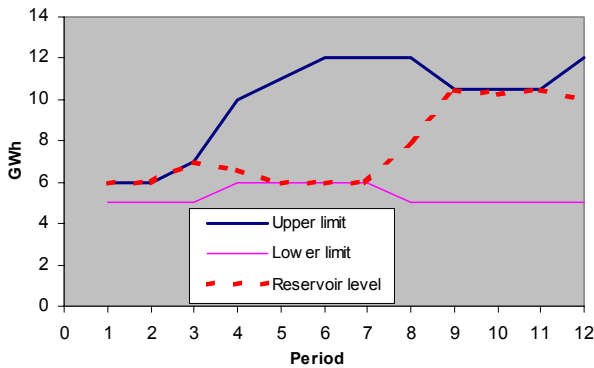


Fig. 1. Reservoir limits and reservoir levels

Table VI also shows that profit water value can be either equal to or different to cost water value. For example in period  $p_5$  both values are equal, which means that GenCo  $x$  maintains its total energy production. On the other hand in period  $p_9$  profit value is different to cost value. By analyzing the results, this can be explained by the existence of a new market equilibrium point, because GenCo  $x$  increments its total production, while GenCo  $y$  decrements it.

## 5. Conclusions

This paper has analyzed the assessment of water resources in competitive environments. In order to deal with liberalized electricity markets, new definitions have been made to adapt traditional water value to the hydro-thermal coordination faced by profit-maximization generation companies. As a result, two different water values have been defined: profit value and cost value. The paper includes a methodology to compute both water values using a market equilibrium approach based on conjectural variations.

The paper studies the meaning of cost water values, and shows how it is the correct economic signal for coordination between medium-term hydro scheduling and short-term hydro operation. All these considerations have been applied to a case study.

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