Cross-border electricity trading modelling: a market equilibrium approach

Félix Fernández-Menéndez[1], Begoña Vitoriano[1], Julián Barquín[1]

[1] Instituto de Investigación Tecnológica
Universidad Pontificia de Comillas de Madrid
C/ Alberto Aguilera 23 – 28015 Madrid (Spain)
Tel.:+34 91 542 28 00, fax:+34 91 542 31 76, e-mail: Felix.Fernandez@iit.upco.es

Abstract. It is widely accepted that generation operation planning can be advantageously modelled through market equilibrium representation. When several areas or countries are intended to function as a whole market, cross-border congested lines require implementation of a management methodology. Of the ones proposed in the literature, this paper is focused on market splitting and explicit auctions.

A model equilibrium approach is presented to represent both congestion management methods through an equivalent optimisation problem. All the analysis is made for a two areas market scenario and mainly focused to the medium term.

The proposed modelling not only allows theoretical studies, but also practical analysis of real-world systems including a detailed representation of their characteristics.

Keywords

Power system economics, interconnected power systems, congestion management, market equilibrium, cross border electricity trading.

1. Introduction

The European electric power system, initially interconnected for reliability reasons, then used also for commercial purposes through well-defined exchange contracts (mostly long term contracts), is now the theatre for a more complex European market. Congestion management has a strong impact on both power system security and market liquidity.

Models to simulate the results of the different proposals for congestion management in the medium-term are crucial to determine how they can affect the generation scheduling in competitive environments.

In Europe, nowadays, are operative several methods to solve cross border transmission limitations [1]. Although very often, ad-hoc procedures have been applied, current trends favour a more systematic approach. In that sense, two of the main alternatives are based on implicit or explicit auctions.

In absence of strategic behaviour, both methods should yield, in equilibrium, the same results. Therefore, adoption of one or another should be based mainly on issues related to its operational convenience. However, existence of market power can lead to very different outcomes.

The aim of this paper is to compute the market equilibrium achieved with the different proposals for congestion management in cross border transactions considering oligopolistic behaviour in the medium-term. With those results it can be determined the actual market power each competitor can exercise in each case, in order to determine which proposals are performing best the objectives they were designed for [2].

There are different approaches to solve the market equilibrium for medium term. One of them is based on solving an equivalent quadratic optimisation problem. This approach has been proved to be suitable to simulate real cases very efficiently [3].

The changes to be made over this model in order to simulate electricity markets assuming both main frameworks of regulation, implicit and explicit auctions, are presented in this paper.

There are two main differences when solving congestions by market splitting or by explicit auction. First, with market splitting each agent have to make a hypothesis about the state of the transmission line (congested or not) and each agent is paid by the production in an area at the price in that area. Second, with explicit auction the auction takes place before clearing the market, so we have two stages. Also in general there will be a price to pay for the use of the interconnection, and each agent is paid by the sales in an area at the price in that area.

The models developed have been applied to a case study. The case study will be focused on the strategic models for each one of the proposals considered, but validated over a perfect market since all the methods to manage the congestions should achieve the same solutions in that case. The analysis is restricted to a two areas case. The results are expected to be very sensitive to assumptions about market design and timing of the market made for each one of the proposals [4].

The sequel of this paper is organised as follows. Next section is devoted to describe the market, placing particular emphasis in the congestion management methods studied. In the third section, the equivalent optimisation models for market splitting and explicit auctions are formulated. These models are applied to a case study in the following section, showing differences in the results for both congestion management methods. Main conclusions are summarised in the last section.

2. Market description.

In the pool market an agent increases the production when the agent can make up for the costs with the price in the market. The costs are not only the production cost but also the decrease in the gains. This is because an increment in the production generates a decrease in the price, which affects to the whole production of the agent.

The pool market is assumed to be an oligopoly. Therefore, agents taking part in the pool market have market power. This means, that the agents can affect the pool price by modifying their sales in the market. How the price varies when each agent modifies his production is a quantity difficult to compute. A popular description is by means of the so-called conjectural variations $\theta_{e,a}$, which is defined as $\theta_{e,a} = -\frac{\partial \lambda_a}{\partial P_{e,a}}$, being λ_a the clearing price for area a and $P_{e,a}$ the production of agent e in area a. The minus sign is written in order to have positive quantities.

Conjectural variations $\theta_{e,a}$ are assumed to be known quantities. They could be derived from market data. For a review see [5].

In this way the market equilibrium is reached when the price equalize the costs:

$$\lambda_a = CM_{e,a} + \theta_{e,a} \cdot P_{e,a} \quad \forall a, e \qquad (1)$$

where,

λ_a: clearing price in area a

$CM_{e,a}$: marginal cost for agent e in area a

$\theta_{e,a}$: conjectural variation

$P_{e,a}$: production of agent e in area a

A. Market splitting

With this method, firstly a power pool price is set according to amounts of demand and generation offered in the whole market area. Then the Transmission System Operator computes a load flow and identifies constrained lines. The whole market area is split into two market areas on both sides of each constrained line. In each new market area a different pool price is defined, and the flows across different market areas being limited to the capacity of the interconnection lines.

With reference to a two areas model, when the market splitting method is implemented to manage congestions, both areas will have the same pool price when the line between them is not congested. In other case each area will have its own pool price.

An important difference between market splitting and explicit auctions is that the energy crossing the border from one area to another has no owner when the market splitting is considered. In this way, the agents are paid for their production in one area at the price in that area. Though, in the explicit auction method the agents are paid for their sales in each area.

In this sense, when the line between the two areas is congested, an agent can only affect the price in an area by increasing or decreasing his production in that area. However, when the cross-border line is not congested and therefore there is only a price for both areas, the agent can affect the price by modifying his production in any of the areas. This fact is represented by the conjectural variations. In the first case there is a conjectural variation per each agent and area. In the latter case there is only one conjectural variation per agent:

1. Line congested: $\theta_{e,a}$
2. Line not congested: θ_e

Therefore, (1) can be seen as expressed for market splitting when the cross-border line is congested. Otherwise, the market equilibrium can be expressed by:

$$\lambda = CM_e + \theta_e \cdot P_e \quad \forall e \qquad (2)$$

where,

λ: clearing price in the whole market (both areas)

CM_e: marginal cost for agent e

θ_e: conjectural variation

P_e: production of agent e in the whole market

B. Explicit auctions.

As mentioned before, when the explicit-auction method is implemented, the energy crossing the border from one area to another has an owner. Before the pool market is cleared, a capacity auction takes place. In this way, each agent has to make two different but related decisions[1]:

1. Capacity to buy in the capacity auction.
2. Production in the pool market.

Both decisions will define the sales for each agent. It is possible that an agent with no physical production can participate in the market. These agents are known as arbitrageurs and they have to make only the first decision.

From the capacity auction will result a price that each agent must pay to cross energy from one area to another. In like manner, from the pool market will result a price per area, the agents will receive for their sales in each area.

In this way, unlike market splitting, the conjectural variation represents how varies the price when each agent modifies his sales instead of his production $\theta_{e,a} = -\frac{\partial \lambda_a}{\partial V_{e,a}}$.

In this sense (1) can be modified to express the market equilibrium with explicit auctions, as follows:

$$\lambda_a = CM_{e,a} + \theta_{e,a} \cdot V_{e,a} \quad \forall e, a \qquad (3)$$

where,

λ_a: clearing price in area a

$CM_{e,a}$: marginal cost for agent e in area a

$\theta_{e,a}$: conjectural variation

$V_{e,a}$: sales for agent e in area a

In a similar way, in the capacity auction an equilibrium is also reached. An agent increases the purchase of capacity when can make up for the costs with the gains.

In this case the gains are the price in the destiny area and the increment of the gains in the origin area because a decrease of sales (or an increase of purchases) produces an increase of the price that affect the whole sales in the origin area.

The costs are the price of the capacity in the auction, the price in the origin area, the decrease in the gains for the sales in the destiny area because an increment of sales produces a decrease in the price, and the increase in the capacity price, which affects all the capacity bought in the auction.

[1] Henceforth, the pool market and the capacity auction will be considered to be simultaneous.

How the capacity price varies when an agent modifies his purchase of capacity is also a quantity difficult to compute. It can be expressed in the form of a conjectural variation $\phi_{e,a,b} = \frac{\partial R_{a,b}}{\partial T_{e,a,b}}$.

Now, the market equilibrium for the capacity auction can be described by the next equation:

$$R = \lambda_b + \theta_{e,a} \cdot V_{e,a} - \lambda_a + \phi_{e,a,b} \cdot T_{e,a,b} - \theta_{e,b} \cdot V_{e,b} \quad \forall e$$

where,

R: capacity price (positive from area a to area b)

$T_{e,a,b}$: capacity purchased for agent e from area a to area b

$\phi_{e,a,b}$: conjectural variation

For the sake of simplicity, it is considered that the agents do not have market power in the capacity auction. Then, the term $\phi_{e,a,b}$, which is the representation of the market power each agent can exert in the capacity price by modifying the capacity purchased in the auction, will be zero. This is a reasonable hypothesis since the capacity auction will be observed by some kind of regulatory entity.

Now, the previous equation can be reformulated as:

$$R = \left(\lambda_b - \theta_{e,b} \cdot V_{e,b}\right) - \left(\lambda_a - \theta_{e,a} \cdot V_{e,a}\right) \quad \forall e \qquad (4)$$

3. Equivalent quadratic optimisation problem

Solving equilibrium models through equivalent optimisation models is not a new idea. The quadratic optimisation models stated in this sections are based on those proposed in [3] and [6], but in this case, modelling also both frameworks of regulation to deal with congestion.

A. Market splitting

When the market splitting method is implemented to manage congestion in the cross-border lines between two areas it is necessary to distinguish whether the line is congested or not. An optimisation problem equivalent to the market equilibrium can be formulated as follows for both cases:

1) Cross-border line not congested

$$\min_{P_e} \sum_{e} \left(C_e(P_e) + \frac{\theta_e \cdot P_e^2}{2} \right)$$

subject to:

$D = \sum_{e} P_e \quad :\lambda$

$P_e \geq 0,\ \theta_e \geq 0,\ D \geq 0,\ C_e \geq 0$

where :

P_e : Production by firm e

$C_e(P_e)$: Cost function of firm e

θ_e : Conjectural variation of firm e

D: Total demand in the whole market (two areas)

λ : Dual variable of demand constraint

Now, the Karush-Kuhn-Tucker optimality conditions can be set out for the previous problem:

$$CM_e + \theta_e \cdot P_e - \lambda = 0 \qquad (5)$$
$$D - \sum_{e} P_e = 0$$

Equation (5) is equal to the market equilibrium condition expressed in (2), and the whole market price in this case can be computed as the dual variable for the demand-production balance constraint.

2) *Cross-border line congested*

$$\min_{P_{e,a}} \sum_{e,a} \left(C_{e,a}(P_{e,a}) + \frac{\theta_{e,a} \cdot P_{e,a}^2}{2} \right)$$

subject to:

$D_a = \sum_{e} P_{a,e} + I \quad :\lambda_a \quad \forall a$

$I - I_{max} \leq 0 \quad :\tau$

$-I + I_{min} \leq 0 \quad :\upsilon$

$C_{e,a} \geq 0,\ P_{e,a} \geq 0,\ \theta_{e,a} \geq 0,\ D_a \geq 0$

and:

$P_{a,e}$: Production by firm e in area a

$C_{e,a}(P_{e,a})$: Cost function of firm e in area a

$\theta_{e,a}$: Conjectural variation of firm e in area a

D_a :Demand in area a

λ_a : Dual variable of demand constraint in area a

I : Cross border flow from area b to area a (positive when the flow is from b to a)

Now, the Karush-Kuhn-Tucker optimality conditions can be set out for the previous problem:

$$CM_{e,a} + \theta_{e,a} \cdot P_{e,a} - \lambda_a = 0 \quad \forall e,a \qquad (6)$$
$$D_a - \sum_{e} P_{a,e} + I = 0 \quad \forall a$$
$$\lambda_a - \lambda_b + \tau - \upsilon = 0$$
$$\tau(I - I_{max}) = 0$$
$$\upsilon(I_{min} - I) = 0$$
$$\tau, \upsilon \geq 0$$

Equation (6) is equal to the market equilibrium condition expressed in (1). As can be deduced, the market price in an area can be computed as the dual variable for the demand-production balance constraint.

B. *Explicit auctions*

For the explicit auction method to manage congestion in the cross-border lines between two areas an optimisation problem equivalent to the market equilibrium can be formulated as follows:

$$\min_{P_{e,a}, T_{e,a,a'}} Z = \sum_{e,a} \left[C_{e,a}(P_{e,a}) + \frac{1}{2} \theta_{e,a} \cdot V_{e,a}{}^2 \right]$$

subject to:

$D_a = \sum_{e} V_{e,a} \ , \forall a \quad :\lambda_a$

$I - I_{max} \leq 0 \quad :\tau$

$I_{min} - I \leq 0 \quad :\upsilon$

$C_{e,a} \geq 0,\ P_{e,a} \geq 0,\ V_{e,a} \geq 0,\ D_a \geq 0$

where

$V_{e,a} = P_{e,a} - \sum_{a' \neq a} \left(T_{e,a,b} - T_{e,b,a} \right) \quad , \forall e,a$

$I = \sum_{e} \left(T_{e,a,b} - T_{e,b,a} \right)$

$T_{e,a,b} \geq 0$

D_a: Demand in area a

$\theta_{e,a}$: conjectural variation of firm e in area a

$V_{e,a}$: Sales for firm e in area a

$T_{e,a,b}$: Exports by firm e, from area a to area b

I: Energy flow between area a and area b

I_{max}: Maximun energy flow from area a to area b

I_{min}: Maximun energy flow from area b to area a

$C_{e,a}$: Cost function for firm e

λ_a: Dual variable of demand contraint in area a

The Karush-Kuhn-Tucker optimality conditions can be set out for the previous problem as:

$$CM_{e,a} + \theta_{e,a} \cdot V_{e,a} - \lambda_a = 0 \ , \forall e,a \qquad (7)$$

$$CM_{e,a} + \theta_{e,b} \cdot V_{e,b} - \lambda_b + \tau - \upsilon = 0 \ , \forall e \qquad (8)$$

$$\mathrm{D}_a - \sum_e V_{e,a} = 0 \ , \forall \mathrm{a}$$
$$\tau \cdot (I - I_{\max}) = 0$$
$$\upsilon \cdot (I_{\min} - I) = 0$$
$$\tau \geq 0, \ \upsilon \geq 0$$

Observing equation (7), expression (8) can be reformulated as

$$\lambda_a - \theta_{e,a} \cdot V_{e,a} + \theta_{e,b} \cdot V_{e,b} - \lambda_b + \tau - \upsilon = 0 \ , \forall \mathrm{e} \qquad (9)$$

The first equation, (7), is equal to the market equilibrium condition (3). As with market splitting, the market price in an area can be computed as the dual variable for the demand-production balance constraint.

Comparing equations (4) and (9), can be easily deduced that:

$$\tau - \upsilon = R \qquad (10)$$

The capacity price is equal to the dual variable of the maximum cross-border-flow constraint that is active. When $\tau \neq 0$ means the flow is from area a to area b, and thus capacity price R is computed as positive. When $\upsilon \neq 0$ means the flow is from area b to area a, and thus capacity price R is computed as negative.

4. Case study

For the analysis of both congestion management methods, market splitting and explicit auctions, the equivalent optimisation problems to both market equilibriums have been programmed. In order to achieve the best clarity in the conclusions, the same sets of data have been used for both models.

The system employed as example has the following characteristics: two areas, four agents, two of which have production in both areas. Meanwhile the other two have production only in one of the areas. The time horizon is divided in 12 periods, which can be assumed to represent the 12 months.

Table 1 shows the demand per area, which has been considered to be inelastic.

Table 1. Demand per area [GW]

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12
Area1	38	40	41	36	35	37	39	41	40	39	38	37
Area2	20	21	20	20	18	17	20	21	20	19	19	18

Table 2 shows the generation distribution of each agent per area.

Table 2. Number of generators and installed power per Area [GW]

		Gen. 1	Gen. 2	Gen. 3
Area 1	Agent 1	17	11	-
	Agent 2	9	11	-
	Agent 3	-	-	5
Area 2	Agent 1	-	-	3
	Agent 3	11	4	-
	Agent 4	6	-	-

In Table 3 is shown the variable cost for each generator considered in the case study.

Table 3. Variable Cost per generator [kEuros/GWh]

	Gen. 1	Gen. 2	Gen. 3
Agent 1	5.15	7.25	12.05
Agent 2	6.05	6.65	-
Agent 3	11.75	13.25	6.35
Agent 4	10.85	-	-

The cross-border line between the two areas has a maximum capacity of 7 GW from Area 1 to Area 2 and 5 GW in the opposite sense.

A. Perfect competition results

First of all, a perfect market scenario has been run for both congestion-management methods. This is made in order to check the goodness of both formulations. The results for both models should be the same, as all the conjectural variations are set to zero.

Table 4 presents the profits each agent perceives for the two frameworks considered. In Table 5 the prices for the market in each area are shown for market splitting (MS) and explicit auctions (EA). Also the capacity price resulted in the capacity auction is shown.

As expected, all the results coincide for both models with the perfect market scenario.

Table 4. Agent's profits for both models assuming perfect competition [kEuros]

	Market splitting	Explicit auctions
Agent 1	419.28	419.28
Agent 2	196.44	196.44
Agent 3	49.91	49.91
Agent 4	63.55	63.55

Table 5. Area's prices for both models assuming perfect competition [kEuros/GWh]

	Area 1 MS	Area 1 EA	Area 2 MS	Area 2 EA	Capacity EA
Avg	7.215	7.215	11.774	11.774	4.56

In the next two subsections, results with an oligopoly scenario will be presented for both congestion-management models.

Both models have been run for two sets of data. The only difference between them is the value of the conjectural variations in Area 1.

From the Data 1 set to the Data 2 set, Agent 1 increases its relative market power. In Table 6 the differences between the two sets of data are shown.

Table 6. Conjectural variations differences from Data1 set to Data2 set [kEuro/GWh].

	Area1	
	Data1 set	Data2 set
Agent 1	0.18	0.25
Agent 2	0.2	0.2
Agent 3	0.18	0.18
Agent 4	0.15	0.15

B. Results for market splitting

The cross-border line is always congested for the two sets of data, from Area 1 to Area 2. As can be seen in Table 7 the increase in the conjectural variation of Agent 1 increases greatly the profits of all the agents with production in Area 1.

Table 7. Agent's profits with market splitting [kEuro]

	Data 1	Data 2	Change %
Agent 1	1387.6	1709.65	23.21
Agent 2	1099.08	1416.44	28.87
Agent 3	338.12	416.37	23.14
Agent 4	159.81	159.81	0

The reason for this increase is shown in Table 8. The price in Area 1 experiences a notable raise when the Agent 1 puts up his relative market power. The price in Area 2, as well as the profit of Agent 4 who has only production in Area 2, does not change.

Table 8. Area's prices [kEuro/GWh]

	Area 1 Data1	Area 1 Data2	Change %	Area 2 Data1	Area 2 Data2	Change %
Avg	10.994	12.298	11.86	13.109	13.109	0

C. Results for explicit auctions

As in the previous case with market splitting, the cross-border line capacity is used completely, from Area 1 to Area 2. However, in this case, as shown in Table 9, the profit of Agent 1 is less sensitive to his conjectural variation change in Area 1.

Table 9. Agent's profits with explicit auctions [kEuros]

	Profit Data1	Profit Data2	Change %
Agent 1	1212.84	1344.04	10.82
Agent 2	872.96	1084.27	24.20
Agent 3	329.83	405.16	22.84
Agent 4	175.99	193.27	9.82

It is curious that the growth in percentage terms for the profit of Agent 1 is much smaller than for the other two agents with production in Area 1. This fact is explained in Table 10, where it is shown that the Agent 1 losses market share in Area 1in favour of Agent 2.

Table 10. Average sales per Area with explicit auctions [GW]

	Area1		Area2	
	Data1 set	Data2 set	Data1 set	Data2 set
Agent 1	18.17	16.63	4.45	5.34
Agent 2	15.25	16.78	4.75	3.21
Agent 3	5	5	4.25	4.87
Agent 4	0	0	6	6

Unlike the market splitting, Table 11 shows that with explicit auctions the price in Area 2 is also sensitive to the change in the conjectural variation value of Agent 1 in Area 1. Even more, a slight change can be seen in the capacity price set in the auction as it is shown in Table 12.

Table 11. Area's average prices [kEuro/GWh]

Area 1 Data1	Area 1 Data2	Change %	Area 2 Data1	Area 2 Data2	Change %
10.53	11.346	7.75	13.336	13.577	1.8

Table 12. Capacity's average prices [kEuro/GWh]

Capacity Data1	Capacity Data2	Change %
4.906	4.944	0.77

D. Market splitting versus explicit auctions

With both data sets, the price in Area 1 is higher when the market splitting method is implemented. Also the agent's profits are substantially higher with market splitting.

Even more, the increase in the price in Area 1 when the market power for Agent 1 increases is quite higher with market splitting. However, when the explicit auction method is implemented the change affects not only the price in Area 1 but also the price in Area 2 and the capacity price.

With market splitting Area 2 seems to be more isolated from what happens in Area 1.Meanwhile for the explicit auction model, a change in the conjectural variation of

Agent 1 in Area 1 affects all the prices and all the agent's sales in both areas.

5. Conclusions

We have presented a quadratic optimisation problem equivalent to the market equilibrium, modelling the different congestion managements methods considered above in the medium-term.

Modelling the congestion management methods proposed and solving a market equilibrium for them give us a valuable information about the actual implications of those methods in the price of the countries affected and the market power exercised by the agents.

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