

Computation of Low-Frequency Electric Fields in Analysis of Electromagnetic Field Exposure

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Abstract

A method for numerical computation of low-frequency electric field near power apparatus and systems based upon the method of moments and the thin-wire approximation is described in this paper. Main feature of the method is a smooth approximation of unknown line charge densities, which is enabled by application of cubic splines. The applicability of the method is illustrated by the analysis of the occupational and general public electric field exposure in substations and in the vicinity of power lines.

Keywords

Electric Field, Integral Equations, Method of Moments, Cubic Splines, Occupational Exposure

1. Introduction

The analysis of the exposures to low-frequency electric field is nowadays an integral part of design in the area of power apparatus and systems. National standards define maximal values of electric field strength for occupational and general public exposure to such a field and design solution must be checked with respect to that criteria.

In the analysis of occupational and general public electromagnetic field exposure near power apparatus and systems we are interested in the steady-state time-harmonic electromagnetic fields at the frequency of 50-60 Hz. At these frequencies the electromagnetic field is a quasi-static field and effect of electric and magnetic fields can be analyzed separately [1].

We study fields in the areas where humans may stay for longer periods (occupational period of stay is eight hours per day and general public period of stay is 24 hours per day). Thus, we are interested only in fields that are far away from sources and therefore we may model sources (charges) as being distributed over one-dimensional lines (thin-wires). We neglect insulators because they influence the electric field only in a near region.

The most effective solution method for such linear and unbounded problems is the application of the method of moments that is based upon the boundary integral formulation for known scalar electric potential of

conductive parts. Geometry of conductive elements of power apparatus and systems is approximated by straight lines and second-order curves. Unknown distribution of charge density in the electric field computation is approximated by cubic splines, and the coefficients of the distribution are determined from known potentials by point matching.

The application of described computational approach is illustrated by the analysis of the occupational electromagnetic field exposure in a 400/110 kV substation and on the intersection of 400 kV and 110 kV transmission lines at the frequency of 50 Hz. Computed fields are compared to the allowed values of electric and magnetic fields that are defined by the Croatian legislation.

2. Numerical calculation

In computation of electric field we deal only with conductors at known potentials. The earth is assumed to be a perfect conductor and we take into the consideration the earth influence by the method of images. The phasor of the scalar electric potential $\phi(\vec{r})$ at any point \vec{r} on the total surface of conductors is related to the phasor of line charge density $\lambda(\vec{r}')$ at any point \vec{r}' on all thin-wire elements (original and image line charges) l by the equation [2]

$$\phi(\vec{r}) - \int_l \frac{\lambda(\vec{r}') dl}{4\pi |\vec{r} - \vec{r}'|} = 0 \quad (1)$$

In order to determine the unknown function $\lambda(\vec{r}')$, the method of moments is applied. The thin-wires are divided into the finite segments Δl_i ($i=1, \dots, N_S$) that may be straight lines and circular arcs. On the i^{th} finite segment we express λ by N_B basis functions t_k as

$$\lambda_i = \sum_{k=1}^{N_B} K_{ik} t_k \quad (2)$$

A cubic distribution ($N_B = 4$) is assumed for λ on each segment and the dependence of the basis functions t_k upon the dimensionless parameter s ($1 \geq s \geq 0$) is

$$t_k = \sum_{j=1}^4 a_{kj} s^{j-1} \quad ; \quad k=1,2,3,4 \quad (3)$$

In order to define the basis functions t_k we use the values of the line charge density and the derivatives of the line charge density at the beginning ($s = 0$) and at the end ($s = 1$) of the segment. The line charge density on each segment is defined by

$$\dot{\lambda}_i(s) = c_1 + c_2 s + c_3 s^2 + c_4 s^3 \quad (4)$$

Thus, the values and the derivatives at the beginning and at the end are:

$$\begin{aligned} \dot{\lambda}_i(0) &= c_1 \quad ; \quad \dot{\lambda}'_i(0) = c_2 \\ \dot{\lambda}_i(1) &= c_1 + c_2 + c_3 + c_4 \quad ; \quad \dot{\lambda}'_i(1) = c_2 + 2c_3 + 3c_4 \end{aligned} \quad (5)$$

The coefficients c_i ($i=1,2,3,4$) are

$$\begin{aligned} c_1 &= \dot{\lambda}_i(0) \quad ; \quad c_2 = \dot{\lambda}'_i(0) \\ c_3 &= -3\dot{\lambda}_i(0) - 2\dot{\lambda}'_i(0) + 3\dot{\lambda}_i(1) - \dot{\lambda}'_i(1) \\ c_4 &= 2\dot{\lambda}_i(0) + \dot{\lambda}'_i(0) - 2\dot{\lambda}_i(1) + \dot{\lambda}'_i(1) \end{aligned} \quad (6)$$

The substitution of the coefficients c_i ($i=1,2,3,4$) into (4) results in

$$\begin{aligned} \dot{\lambda}_i(s) &= \dot{\lambda}_i(0)(1 - 3s^2 + 2s^3) + \dot{\lambda}'_i(0)(s - 2s^2 + s^3) \\ &+ \dot{\lambda}_i(1)(3s^2 - 2s^3) + \dot{\lambda}'_i(1)(-s^2 + s^3) \end{aligned} \quad (7)$$

Accordingly, the coefficients a_{kj} in (3) are contained in the matrix

$$[A] = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad (8)$$

The coefficient K_{i1} in (2) is the value of the line charge density at the beginning of the i -th segment, K_{i2} is the derivative of the line charge density at the beginning of the i -th segment, K_{i3} is the value of the line charge density at the end of the i -th segment, K_{i4} is the derivative of the line charge density at the end of the i -th segment. The substitution of (2) into (1) results in the linear integral equation

$$\dot{\phi}(\vec{r}) = \sum_{i=1}^{N_s} \left\{ \sum_{k=1}^2 L_{ik} \int_{\Delta_i} \frac{t_k(\vec{r}') dl}{4\pi |\vec{r} - \vec{r}'|} + \sum_{k=1}^2 L_{i+1,k-2} \int_{\Delta_i} \frac{t_k(\vec{r}') dl}{4\pi |\vec{r} - \vec{r}'|} \right\} \quad (9)$$

We derive a linear equation system for unknown complex coefficients L_{ik} by point matching at the points defined by $s = \frac{1}{3}$ and $s = \frac{2}{3}$ on each segment. The line integrals in (9) are computed numerically using a globally adaptive scheme based on Gauss-Kronrod rules. The solution of the system determines the differentiable approximation of the line charge λ on each segment. Afterwards, by the usage of the expression

$$\begin{aligned} \dot{E}(\vec{r}) &= \sum_{i=1}^{N_s} \left\{ \sum_{k=1}^2 L_{ik} \int_{\Delta_i} \frac{t_k(\vec{r}')(\vec{r} - \vec{r}') dl}{4\pi |\vec{r} - \vec{r}'|^3} + \right. \\ &\left. + \sum_{k=3}^4 L_{i+1,k-2} \int_{\Delta_i} \frac{t_k(\vec{r}')(\vec{r} - \vec{r}') dl}{4\pi |\vec{r} - \vec{r}'|^3} \right\} \end{aligned} \quad (10)$$

we may calculate the vector-phasor of the electric field strength at any point \vec{r} . Such vectors are elliptically polarized and their magnitude varies with respect to time. We use the effective value of the magnitude

$$\begin{aligned} E_e &= \sqrt{\frac{1}{T} \int_0^T [E_x^2(t) + E_y^2(t) + E_z^2(t)] dt} = \\ &= \sqrt{\frac{E_{x,\max}^2 + E_{y,\max}^2 + E_{z,\max}^2}{2}} \end{aligned} \quad (11)$$

as an equivalent value of elliptically polarized field vectors in studies of electromagnetic field exposure.

3. Analysis of Electric Field Exposure

The International Commission on Non-Ionizing Radiation Protection (ICNIRP) issued the Guidelines [3] for limiting the electromagnetic field exposure up to 300 GHz. The analysis of the electric field exposure may be divided into two parts:

A. Analysis of the occupational exposure

The analysis of the occupational exposure deals with healthy adults who are aware of the risk and who are likely to be subject to medical surveillance. The recommended occupational reference level for the frequency of 50 Hz is $E_{ref} = 10000$ V/m. The accepted limit value for occupational exposure in Croatia is $E_l = 5000$ V/m.

B. Analysis of the general population exposure

The analysis of the occupational exposure has to be based on "broader considerations such as health status, environmental conditions, special sensitivities, possible effects on the course of various diseases, as well as limitations in adaptation to environmental conditions and responses to any kind of stress in old age". The influence of these considerations is insufficiently explored, and the limits for general population exposure have to be considerably smaller. The recommended general public reference level for the frequency of 50 Hz is $E_{ref} = 5000$ V/m. The accepted limit value for general public exposure in Croatia is $E_l = 2000$ V/m.

As an example of the application of the described procedures we analyze the occupational exposure in a 400 kV part of a substation that consists of five 400 kV lines and two power transformers. The model of the substation is shown in Fig. 1.

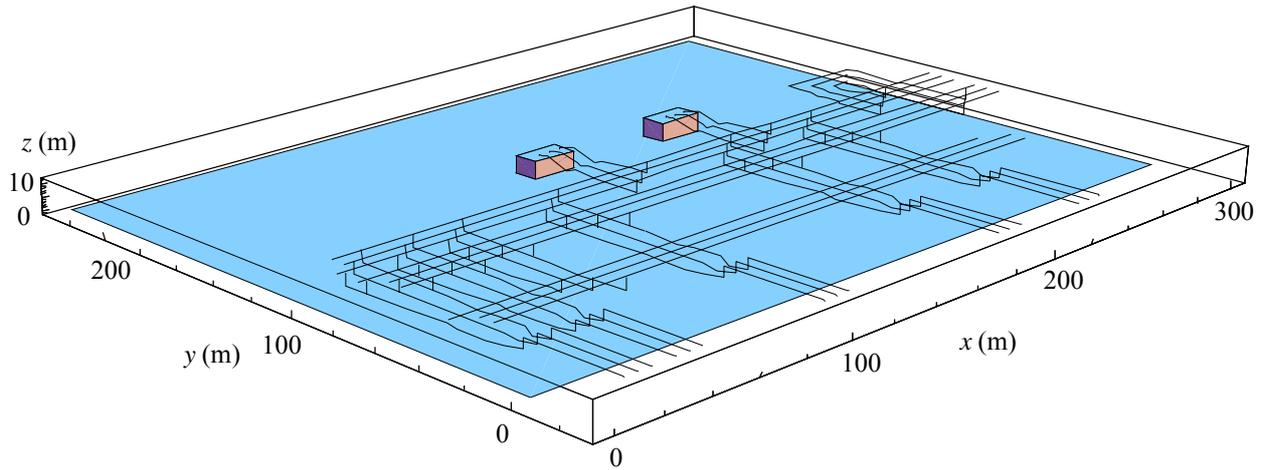


Figure 1. Model of the 400 kV part of a substation

The computation of the electric field is defined by known potentials of wires, which are equal to phase-to-ground voltages of symmetrical three-phase system. As the limit value of occupational exposure is defined in the case of a homogeneous field, we have to calculate the mean value

of the effective value of the magnitude of the electric field between the ground and the height of 2 m.

The results are shown in Fig. 2.

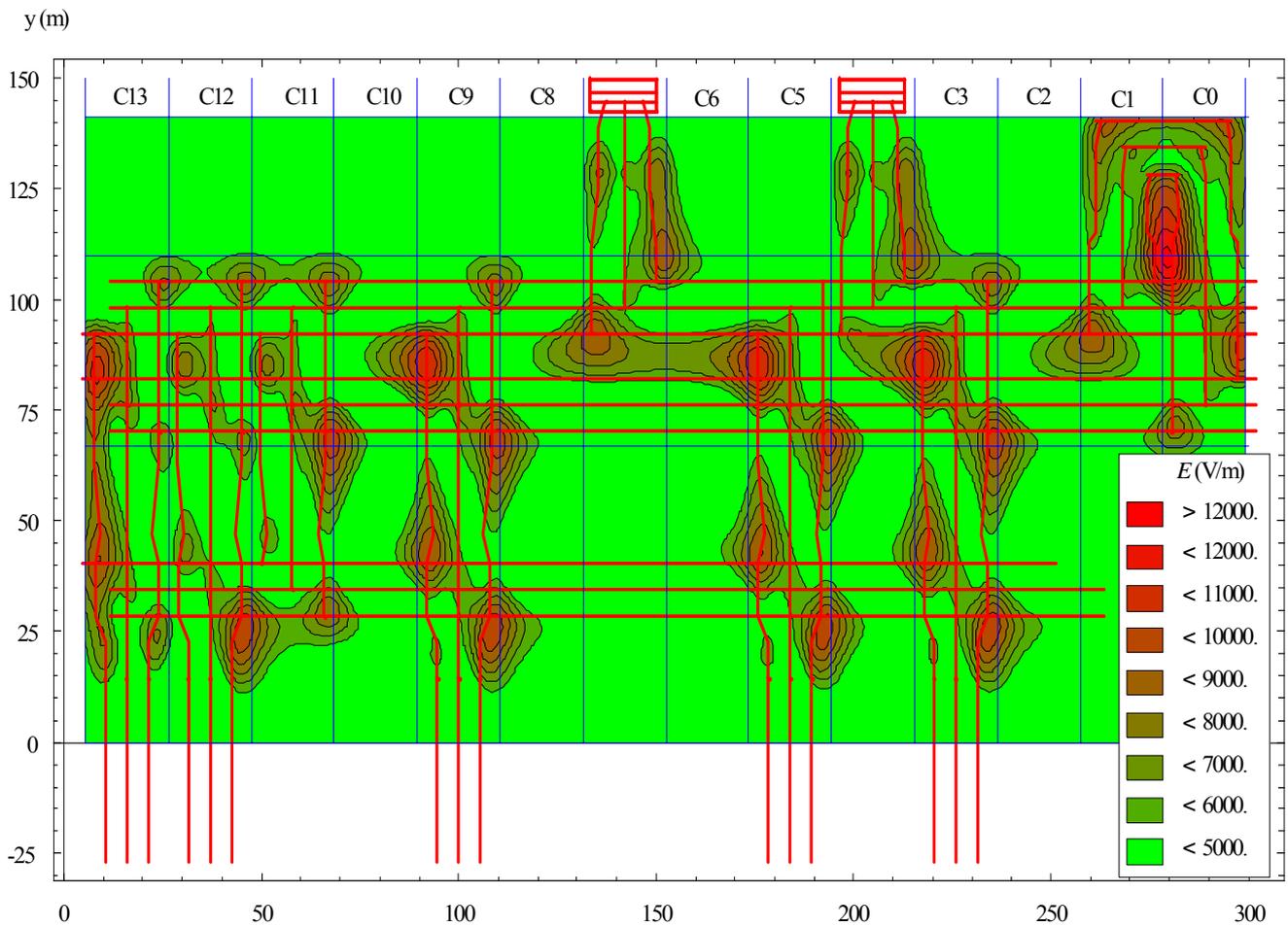


Figure 2. Areas of occupational stay

Another example taken into consideration is the intersection of 400 kV and 110 kV transmission lines. Effective value of electric field is calculated on rectangular plane 2 m above ground (Fig.3.)

Results of the calculation are shown in Fig.4. Maximal values of electric field strength is $E_{\text{eff,max}} = 3300 \text{ V/m}$, therefore analysis has to be done for general public exposure.

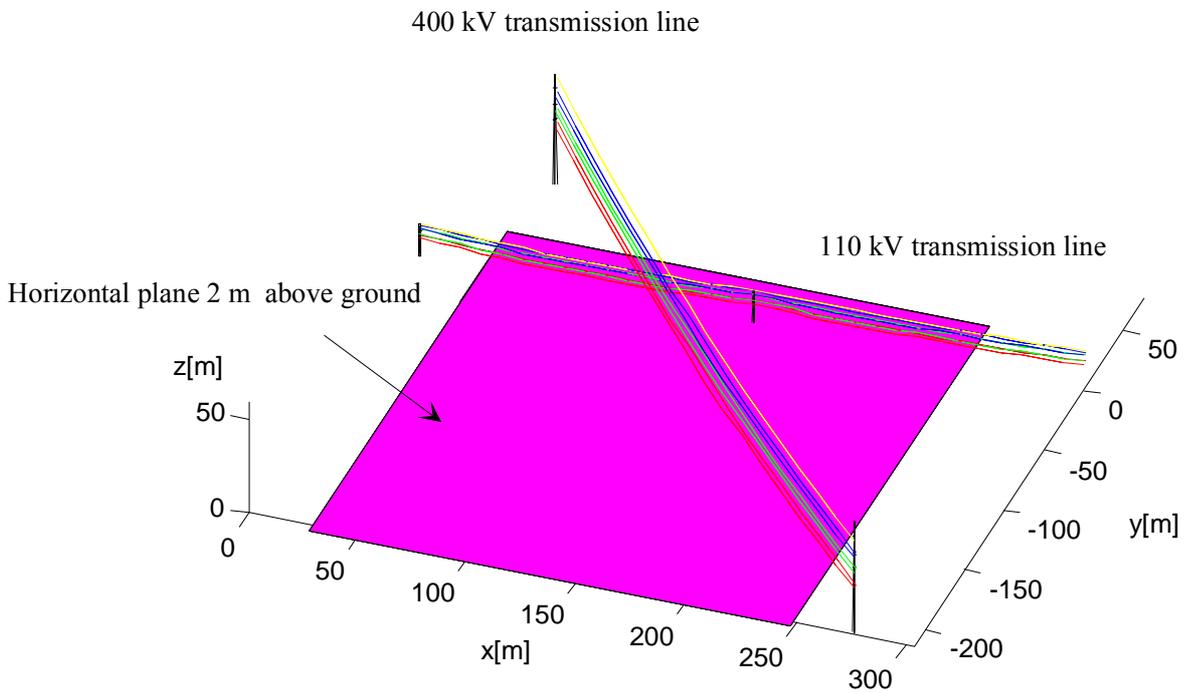


Figure 3. Intersection of transmission lines

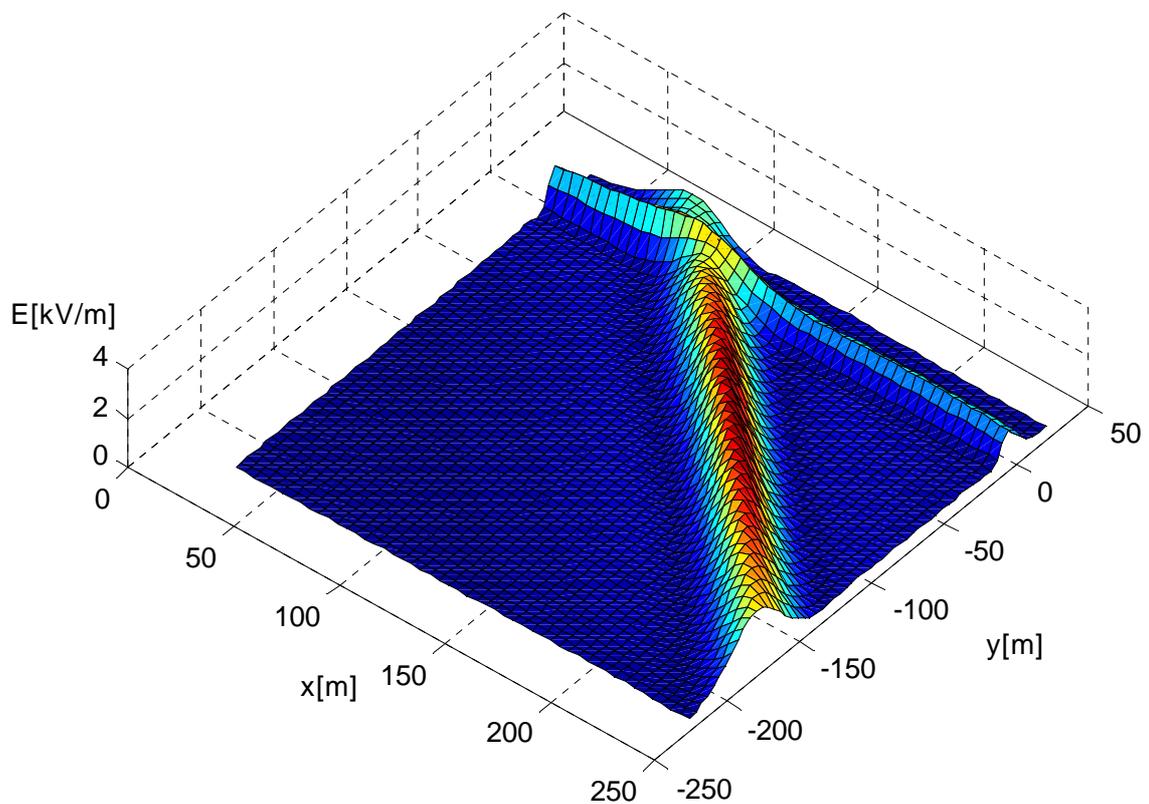


Figure 4. Electric field strength 2 m above ground

Final result of the analysis is the determination of the area in which the general public exposure in duration of 24 hours per day is forbidden. This is the area where effective value of the electric field strength exceeds 2000 V/m. This area is shown in Figure 5.

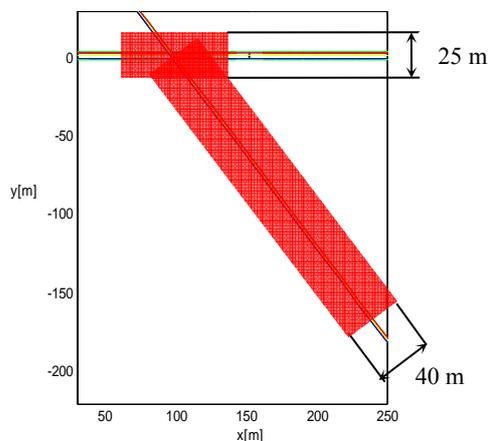


Figure 5. Area in which is general public exposure forbidden

4. Conclusion

A numerical method for computation of low-frequency electric field near power apparatus and systems that may be used for analysis of occupational and general public field exposure has been proposed. The computation is based on the numerical solution of the boundary integral equations by point matching technique and may be easily applied to any complex three-dimensional geometry.

References

- [1] Haznadar, Z. and Štih, Ž. *Electromagnetic Fields, Waves and Numerical Methods*, IOS Press, Amsterdam 2000, pp 359-388
- [2] Štih, Ž. "High voltage insulating system design by application of electrode and insulator contour optimization" *IEEE Trans. on Electrical Insulation*, vol. 21, pp 579-584, Aug. 1986.
- [3] ICNIRP: *Guidelines for limiting exposure to time-varying electric, magnetic, and electromagnetic fields (up to 300 GHz)*, *Health Phys.* 75(4), pp 494-522, 1998.