Levitation Heating as a Multiparametric Coupled Problem

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Abstract. Levitation heating of electrically conductive nonmagnetic bodies (as the first step of their levitation melting) represents a relatively complicated process characterized by mutual interaction of electromagnetic and temperature fields and possibly also by eventual movement of the heated body. Its mathematical model consists of a system of partial and ordinary (generally nonlinear) differential equations whose coefficients are temperature-dependent functions. The velocity and overall efficiency of heating depends on a lot of parameters such as geometry of the inductors, shape of the heated body, amplitudes and frequency of the field currents etc. The paper deals with detailed analysis of a two-inductor axisymmetric arrangement with the aim to optimize its configuration and accelerate the heating as much as possible. The problem is solved in the quasi-coupled formulation. The theoretical results are illustrated on an example.

Key words

Levitation heating, electromagnetic field, temperature field, coupled problem, numerical analysis.

1. Introduction

Levitation heating usually represents the first step of the process of melting of strongly reactive metals or alloys (such as some titanium aluminides) that are used in space research, chemistry, medicine etc. The main purpose of this technology is to prevent the mentioned materials from contacting with crucible walls and their eventual contamination, see, for example [1], [2] and [3].

The levitating system consists of one or more inductors producing time varying magnetic field. Eddy currents induced in the processed metal body inserted into this field generate in it both the Joule losses and Lorentz forces. The workpiece is lifted into a stabilized position characterized by the balance of the Lorentz and gravitational forces, heated and then molten.

As the process requires a lot of energy, the crucial point of designing the system is to make it as efficient as possible. The efficiency is a function of the shape and mutual position of the field coils, shape of the heated body and parameters of the field currents (amplitude, frequency and eventual phase shift).

Some problems associated with the process were already solved and published. An analytical way of modeling of levitation of a nonferromagnetic sphere in a system of idealized inductors (mainly from the viewpoints of the force effects and instabilities) was used, for instance, in [4] and [5]. But the investigated arrangements are very simple, rather unrealistic and the most valuable result is probably a newly developed analytical method of solution working with special gauging of potentials. Other methods based on the analytical approach in simplified arrangements are described in [6] and [7].

Numerical algorithms of solution were also applied and can be found in references (see, for example, [8] and [9]). Investigated were both force effects and temperature rise of the heated body. But the electromagnetic and thermal effects were often explored separately, regardless the mutual interaction of the corresponding quantities.

The paper investigates the influence of the most significant parameters of a levitating system on its operation characteristics. The system consists of two conical coils and a body whose mass is always the same, but as for its shape it can be a sphere, cylinder (or also truncated cone). The theoretical analysis is illustrated on an example whose results are discussed.

2. Formulation of the technical problem

Consider an arrangement depicted in Fig. 1. The body **3** (part **a**) lies at the bottom of the lower coil **1** (but, generally, it may lie at any position). This starting position is denoted by letter **S**. At the time $t = 0$ both coils are connected to the sources of harmonic current. The first one carries current of amplitude I_1, the second one I_2 (and these amplitudes are supposed to be known and constant during the whole process), both of frequency f. These currents start producing harmonic magnetic field whose distribution also depends on mutual phase shift of both currents. The field induces eddy currents of densities J_{eddy} in body **3.** The body starts to be heated and also moves up due to electrodynamic forces. After finishing the transient, the body finds the final stabilized end position **E** (part **b** of Fig. 1) characterized by the balance of the Lorentz and gravitational forces. At this position the body continues to be heated up to its melting temperature T_{M}. Of course, heating leads to variations of its physical parameters (including electrical conductivity) that may affect the mentioned end position. On the other hand, this change was found to be practically negligible.

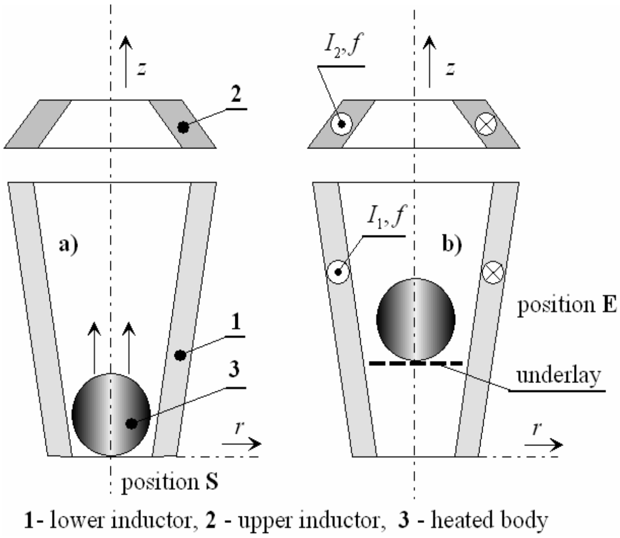

Fig. 1: The investigated arrangement

The aim of the paper is to find the dependence of velocity of heating on the

- degree of taper of both conical field coils,
- shape of the body (whose mass is supposed always the same).

3. Mathematical model and its solution

The mathematical model of the problem generally consists [10] of two partial differential equations describing distribution of harmonic electromagnetic and nonstationary temperature fields and one nonlinear ordinary differential equation describing the motion of the body.

The definition area of electromagnetic field comprises the whole system containing several subdomains (described in Fig. 2) and is bounded by axis z and artificial boundary sufficiently distant from the system. As the system is linear (in our case it contains no ferromagnetic parts) and velocity of the mechanical motion small, the electromagnetic field distribution may generally be described by the Helmholtz equation for the phasor of vector potential $\boldsymbol{A}$ in the form [11]

$$\mathrm{curl}\,\mathrm{curl}\,\underline{\boldsymbol{A}} + \mathrm{j}\cdot\mu_0\omega\gamma\underline{\boldsymbol{A}} = \mu_0\underline{\boldsymbol{J}}_{\mathrm{ext}} \tag{1}$$

where γ denotes the electrical conductivity (a function of temperature T), ω the angular frequency of the field currents and $\underline{\boldsymbol{J}}_{\mathrm{ext}}$ their density. As the arrangement is axisymmetric, vector potential and densities of field and eddy currents have only one nonzero component (A_{φ}, $J_{\mathrm{ext},\vartheta}$ and $J_{\mathrm{eddy},\vartheta}$, respectively) in the circumferential direction ϑ_0.

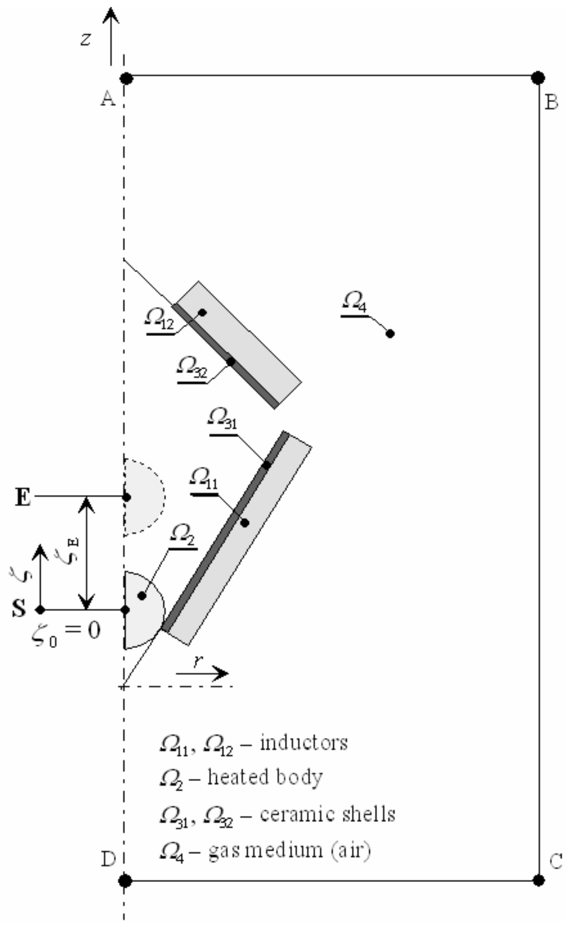

Fig. 2: Definition area of electromagnetic field

As for the boundary conditions, the value of vector potential $\boldsymbol{A}$ along the whole boundary ABCDA is equal to zero (axis AD – antisymmetry, artificial boundary ABCD represents a force line with continuity of potential at points A and D).

Eddy current densities and specific average Joule losses in the heated body are given by formulas

$$\underline{J}_{\mathrm{eddy},\vartheta} = \mathrm{j}\cdot\omega\gamma\underline{A}_{\vartheta}, \tag{2}$$

$$p_{\mathrm{J}} = \frac{\underline{J}_{\mathrm{eddy},\vartheta}\cdot\underline{J}^{*}_{\mathrm{eddy},\vartheta}}{\gamma} \tag{3}$$

where $\underline{J}^{*}_{\mathrm{eddy},\vartheta}$ is the complex conjugate to $\underline{J}_{\mathrm{eddy},\vartheta}$. The total Lorentz force acting on the body is given by integral

$$F_{\mathrm{Lz}} = -\int_{V_3}\left(\underline{J}_{\mathrm{eddy},\vartheta}\cdot\frac{\partial\underline{A}^{*}_{\vartheta}}{\partial z}\right)\mathrm{d}V \tag{4}$$

where V_3 is the volume of the heated body.

The nonstationary temperature field is calculated only inside the heated body. The Fourier-Kirchhoff equation describing its distribution reads [12]

$$\mathrm{div}\left(\lambda\cdot\mathrm{grad}\,T\right)=\rho c\cdot\frac{\partial T}{\partial t}-p_{\mathrm{J}} \qquad (5)$$

where λ denotes the thermal conductivity, ρ the specific mass of the heated material, c its specific heat and p_{J} is given by (3). The boundary conditions are as follows:

- axis of symmetry $\quad \dfrac{\partial T}{\partial n}=0$,
- otherwise $\quad -\lambda\cdot\dfrac{\partial T}{\partial n}=\alpha\cdot\left(T-T_{\mathrm{ext}}\right)+\sigma C\left(T^4-T_{\mathrm{i}}^4\right)$

where α is the coefficient of convective heat transfer, σ the Stefan-Boltzmann constant, T_{ext} the temperature of ambient gas, C the modified coefficient of emissivity (that includes the shape of the body and eventual manifold reflections) and T_{i} the average surface temperature of the inductors.

Finally, the movement of the body is described by a strongly nonlinear ordinary differential equation for velocity in direction z

$$m\cdot\frac{\mathrm{d}v_z}{\mathrm{d}t}=F_{\mathrm{L},z}-mg-c_{\mathrm{x}}\cdot\frac{\rho_{\mathrm{m}}Sv_z^2}{2}, \qquad (6)$$

m being the mass of the body, S its maximum cross-section, ρ_{m} the density of the ambient medium, c_{x} the aerodynamic coefficient (a function of the Reynolds number) and $F_{\mathrm{L},z}$ the total Lorentz force acting on the body that is given by integral (4).

The initial condition reads $v_z\left(0\right)=0$ and the trajectory ζ measured from the starting position **S** (see Fig. 2) can easily be determined by integration of velocity

$$\zeta=\int_0^t v_z\cdot\mathrm{d}\tau,\ \ \zeta\left(0\right)=0. \qquad (7)$$

The mathematical model expressed by equations (1), (5) and (7) with the corresponding boundary and initial conditions was solved by the FEM-based professional code QuickField 5 supplemented with a lot of single-purpose user procedures developed and written by the authors mostly in Matlab-SIMULINK.

4. Illustrative example

We solved an arrangement depicted in details in Fig. 3 for a number of alternatives differing in several important parameters. Both field coils contained the same number of turns, but differed by the degree of tapering (see Fig. 3, angles β and φ). Different were also the shapes of the heated body.

A. Input data

- Geometry: $r_1=0.055$ m, $l=0.178$ m, $N_1=36$, $N_2=18$, $s_1=0.011$ m, $s_2=0.01$ m, $\delta=0.052$ m.
- Field currents: $J_1=4.67\cdot10^7$ A/m^2, $J_2=6.67\cdot10^7$ A/m^2, $f=5$ kHz, no phase shift between them.

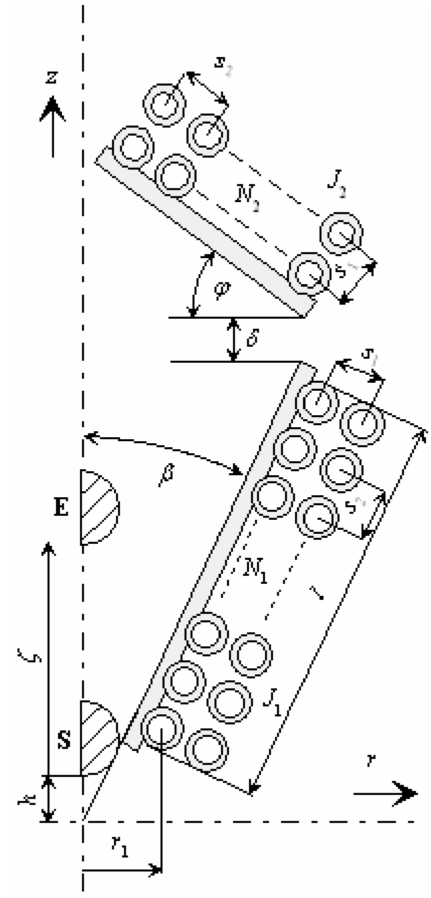

Fig. 3: The solved configuration

- Field coils: made from a hollow copper (Cu 99) conductor (internal diameter 4 mm, external diameter 8 mm), intensively cooled by water. Their electrical conductivity $\gamma=5.7\cdot10^7$ S/m, relative permeability $\mu_{\mathrm{r}}=1$.
- The heated bodies are aluminum workpieces (Al 99.5, $\gamma=3.4\cdot10^7$ S/m, $\lambda=229$ W/mK, $\rho=2700$ kg/m^3, $c=896$ J/kgK) of mass $m=1$ kg and volume $V=0.00037037$ m^3. Temperature $T_{\mathrm{ext}}=20$ °C, $T_{\mathrm{max}}=650$ °C, $\alpha=20$ W/m^{2}K. The workpieces are shaped, in turn, as a sphere, cylinder and truncated cone.
  - Sphere: radius $R=0.04455$ m.
  - Cylinder: radius $R=0.03892$ m, height (that is supposed to be equal to $2R$) $H=0.07884$ m.
  - Truncated cone: smaller radius $R_1=0.03$ m, larger radius $R_2=0.0474$ m, height (that is supposed to be equal to its mean diameter) $H=0.0774$ m.
- Variable are the angles β and φ (see Fig. 3).
- Radiation was neglected at this stage of research. There are principally no problems with the implementation of the corresponding boundary condition itself; complicated is, however, determination of coefficients respecting the influence of geometrical shape, multiple reflections and some other quantities. The associated problems will be solved in the frame of next research in the domain.

B. Computations

Electromagnetic field: computations were carried out on a mesh with about 250000 elements. Carefully was tested the geometrical convergence of results and position of the artificial boundary in order to reach accuracy at the level of three valid digits.

The nonstationary temperature field was calculated on a mesh with about 25000 elements (covering only the heated body), which assured comparable accuracy. The time step was 0.1 s.

The dynamic characteristic was calculated only for three different cases. It was shown that the movement of the body is damped only very slowly and its getting into the stabilized position takes a long time (tens of seconds). More information is given in the next paragraph.

C. Selected results – sphere

First we investigated the mechanical transient whose parameters depend on the initial position of the heated body. Its starting position was at the bottom of the lower coil for degrees of tapering $\beta = 30°$ and $\varphi = 0°$. On the basis of a series of electromagnetic computations we constructed the static characteristic of the system (dependence of the Lorentz force on lift ζ. It is depicted in Fig. 4 (the balanced position being $\zeta_0 = 0.1\,\text{m}$). From solution of (6) we found that the drag force represented by its third term is for real velocities very small (at the level of 0.05% of the total Lorentz force), see Fig. 5. That is why the system behaves as a weakly damped oscillator with period 0.086 s, which follows not only from Fig. 5 but also from Fig. 6 showing the time evolution of velocity v_z within the first period of oscillation.

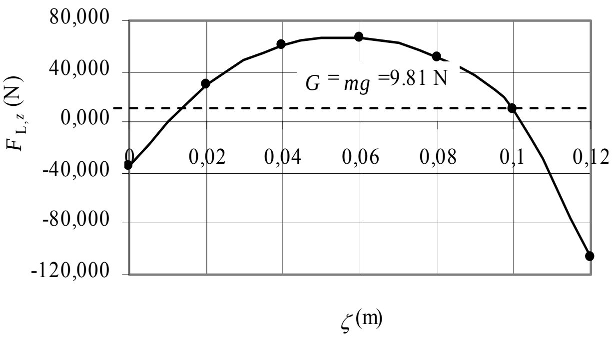

Fig. 4: Static characteristic of the system for $\beta = 30°$ and $\varphi = 0°$ (sphere)

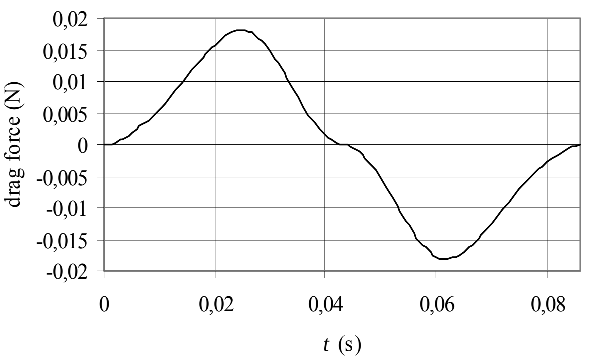

Fig. 5: Time evolution of the drag force in the first period of oscillation for $\beta = 30°$ and $\varphi = 0°$ (sphere)

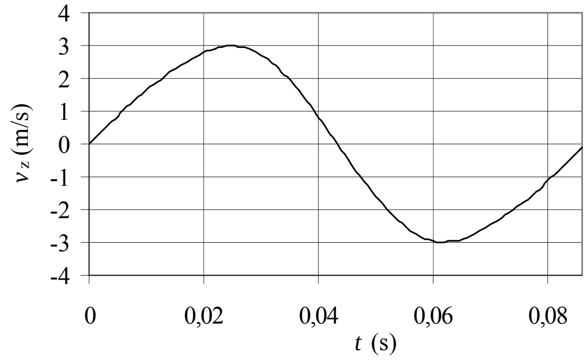

Fig. 6: Time evolution of velocity of the sphere in the first period of oscillation for $\beta = 30°$ and $\varphi = 0°$

Similar dependencies were obtained also for other solved arrangements of the system. But generally, these oscillations substantially deteriorate the parameters of the process of heating. First, the real system is usually not quite axisymmetric and even small deflection from symmetry could result in undesirable 3D oscillations (that are well observable during real experiments before the body is molten). Second, the efficiency of heating decreases because of growth of the coefficient of convective heat transfer α appearing in the boundary condition for temperature field. The more efficient way of heating of the body, therefore, obviously consists in preliminary computational determination of its balanced position and putting it there, for example, on a ceramic underlay (that can be removed later), as is depicted in Fig. 1. In real arrangements, nevertheless, we are not able to completely avoid the oscillations, but they can be reduced significantly.

That is why all next examples are solved under the condition that the process of heating starts at the stabilized position of the body characterized by the equilibrium of the Lorentz and gravitational forces.

Next series of computation was aimed at determination of the time evolution of the average temperature T_{ave} of the heated body in balanced position for various values of angles β and φ. The results are summarized in Fig. 7.

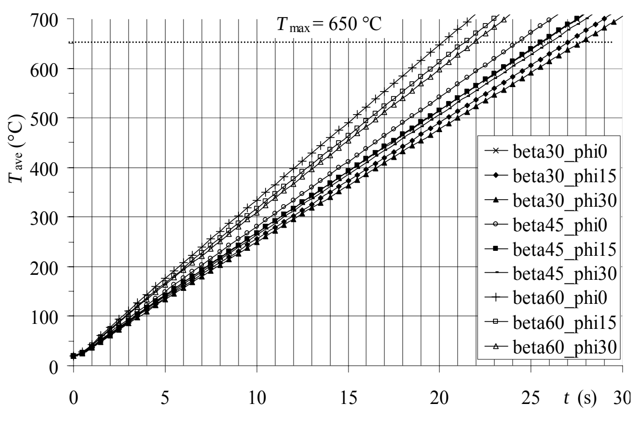

Fig. 7: Time evolution of the average temperature of the sphere for various angles β and φ

The dependencies are almost linear. The shortest time of heating (about 20.1 s) was obtained for angles $\beta = 60°$

and $\varphi = 0°$. The static characteristic of this system (with somewhat higher Lorentz forces than in Fig. 4 and balanced position $\zeta_0 = 0.086$ m) is depicted in Fig. 8.

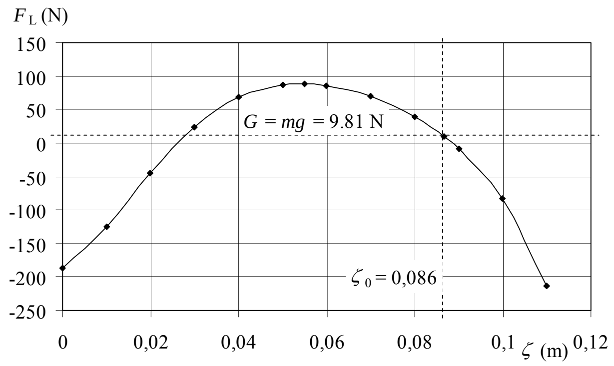

Fig. 8: Static characteristic of the system for $\beta = 60°$ and $\varphi = 0°$ (sphere)

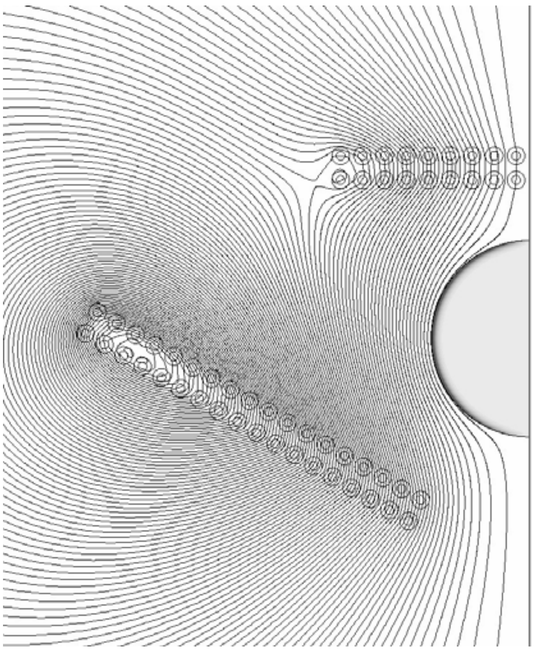

Fig. 9: Magnetic field in the system for $\beta = 60°$ and $\varphi = 0°$ (sphere)

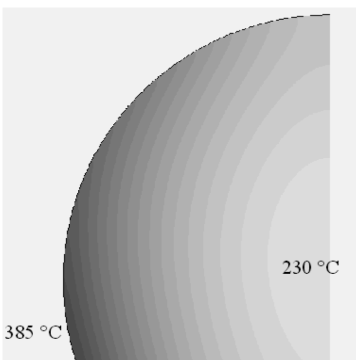

Fig. 10: Distribution of temperature in the sphere for $\beta = 60°$ and $\varphi = 0°$, $t = 9$ s, $T_{ave} = 300$ °C

Fig. 9 shows distribution of magnetic field in the system, near the coils. Eddy currents (as sources of heat) produced in the sphere are distributed only in its surface layer (dark grey color). Fig. 10 shows distribution of temperature after 9 s of heating when the average temperature reaches 300 °C. The difference between the temperatures near the centre and in the surface layers reaches about 155 °C. Development of the temperature distribution in the later phases of heating is somewhat similar as in Fig. 10, but the difference between the highest and lowest temperatures reduces. This follows from good thermal conductivity of aluminum.

In fact, at the moment when the average temperature of the sphere reaches 650 °C, its surface layers start to melt (their temperature may exceed the melting point of aluminum, which is about 660 °C.

Analogously, we calculated similar characteristics for a cylindrical body. Fig. 11 shows the time evolution of its average temperature T_{ave} in balanced position, again for various values of angles β while $\varphi = 0$ (there is no need to carry out further computations for $\varphi > 0$ because nonzero angle φ prolongs the period of heating, as shown in case of the sphere in Fig. 7).

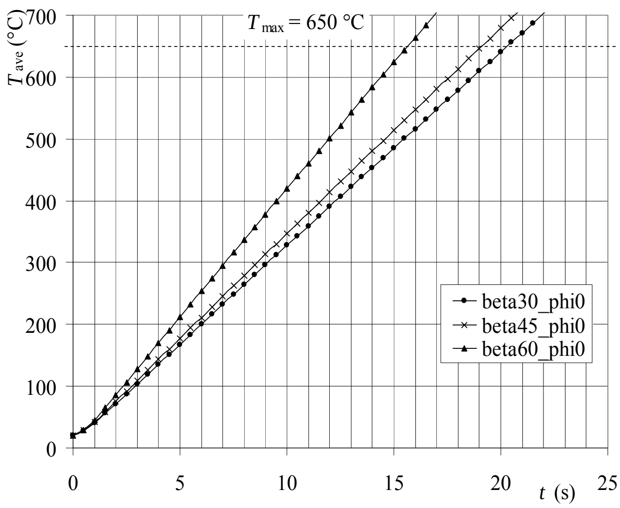

Fig. 11: Time evolution of the average temperature of the cylinder for various angles β and $\varphi = 0$

The time of heating is now somewhat shorter. For $\beta = 60°$ the time of reaching $T_{ave} = 650$ °C is approximately 15.6 s. The static characteristic of the system with cylinder for $\beta = 60°$ and $\varphi = 0$ is depicted in Fig. 12. Now the equilibrium position is given by $\zeta = 0.082$ m.

The dependence of the total Joule losses P_J on ζ for the heated body of all three shapes (and for $\beta = 30°$ and $\varphi = 0°$) is depicted in Fig. 13. The value P_J is obtained by integration of (3) over the volume of the body. And Table I contains the corresponding parameters of heating. It is clear (and it was proved even for other values of angles β and φ) that it is the cylinder whose heating takes the shortest time. As for the cone, the time of heating depends on its degree of taper that represents another degree of freedom in this case.

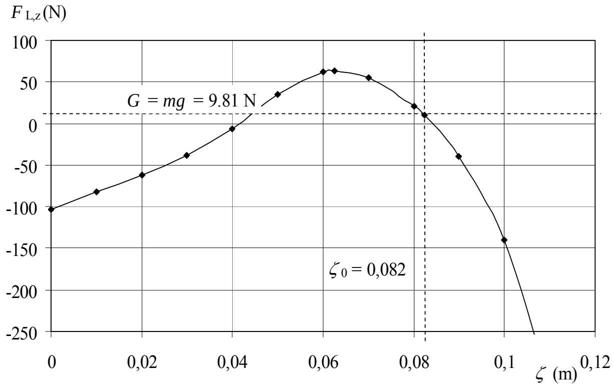

Fig. 12: Static characteristic of the system for $\beta = 60°$ and $\varphi = 0°$ (cylinder)

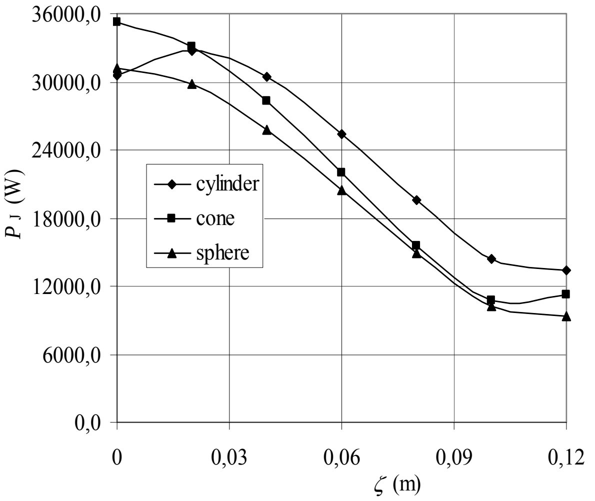

Fig. 13: Dependence of the total Joule losses P_J produced in the body on ζ (for $\beta = 30°$ and $\varphi = 0$)

TABLE I – Balance positions, Joule losses and times t_{650} (time when the average temperature of the body reaches temperature $T_{max} = 650$ °C) for $\beta = 30°$ and $\varphi = 0$

Body	Balance position ζ_0 (m)	Total Joule losses (W)	Time t_{650} (s)
cylinder	0.1003	29808	19.0
cone	0.0919	29681	19.1
sphere	0.1000	23493	24.2

5. Conclusion

Levitation heating is a complex process whose time evolution depends on a lot of parameters (geometry of the inductors, shape of the heated body and parameters of the field currents). The paper shows how the results depend on several of them and provides a lot of related information and characteristics. A considerable attention has been paid to dynamic phenomena (oscillation of the body due to interaction of the Lorentz, gravitational and drag forces). These phenomena are undesirable because of deceleration of heating due to higher convection of heat and, generally, instabilities as no heating system is perfectly axisymmetric.

From the viewpoint of the velocity of heating, a cylinder seems to be the most advantageous body. But the differences are not too high (see Tab.1) and so far it is not quite clear what effects are caused by simplification of the model with respect to geometry and neglecting of radiation.

Next research in the field will be aimed at investigation of the influence of radiation that has decelerating effect on the velocity of heating.

Acknowledgment

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