Main inductance determination in rotating machines. Analytical and Numerical calculation: A didactical approach


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Abstract.

The accurate determination of saturated magnetizing inductances has been the subject of much research over a long time. These results are necessary for the appropriate adjustment of control regulation loops and for the improvement of transient response and stability of electric drives. Traditionally the analytical calculation involves the determination of some empirical factors, such as the d-axis and q-axis reactance factors. In references [1] to [3] there are many expressions for salient and non-salient pole machines, but these are valid only for the considered pole shape.

If possible we should use an expression, or method, independent of the pole shape. Analytical formulation is not adequate for this reason. Now we can use the FE method to calculate this and other parameters.

In addition the time devoted today to the design of electrical machines has been reduced and this makes it impossible to use a lot of empirical or graphical methods. The use of FEM provides a way to quickly and accurately calculate the size of an electrical machine and its parameters. This paper has been written to describe this methodology in an educational environment.

Keywords.

Main inductance determination, FE method, cylindrical and salient pole machines.

1. Analytical calculation of magnetizing inductances.

In the following paragraphs we describe how obtain an analytical expression for the main inductances. This methodology shows how these are function of the pole shape and how explain this in an educational environment.

A. Uniform air gap machine

The magnetizing inductance of a uniform air gap machine can calculated according to the following procedure:

Calculation of:

A. MMF created to the 3-phase equilibrate current system

B. Air gap induction \( B_1 \) (only considers the fundamental component)
C. Total flux per phase \( \Phi \)
D. Main inductance determination: \( L_m = \Phi / I \)

The following expression is the result of this process.

\[
L_m = \frac{\mu_0}{\pi} \cdot m \cdot \frac{D \cdot L}{g_{eq}} \left( \frac{N \cdot \xi}{p} \right)^2
\]

where: \( m \) – number of phases, \( D \) – air gap diameter, \( L \) – length of the machine, \( g_{eq} \) – equivalent air gap (with Carter’s and saturation correction), \( N \) – number of turns per phase, \( p \) – pole pairs.

B. Salient pole machines

The calculation is similar, but we found some differences:

- We calculate the MMF projection over two axis: direct and quadrature axis.
- Thus we determine the induction create for these two components and determine the fundamental component.
- Thus we can calculate the flux and the main inductance for every component:

\[
L_{md} = \Phi_d / I; L_{mq} = \Phi_q / I
\]

These process leads to the following expressions:

\[
L_{md} = k_d \cdot L_m; \quad L_{mq} = k_q \cdot L_m
\]

Where \( L_m \) is the magnetizing inductance calculate supposing that the air gap is uniform and \( k_d \) and \( k_q \) are coefficients that depended on pole shape.

The following table shows these coefficients for different pole shape configuration; the first row is for a classical salient pole synchronous machine and the others are for permanent magnet machines.
Table 1. direct and quadrature correction factors.

<table>
<thead>
<tr>
<th>Machine Type</th>
<th>Formulas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salient pole synchronous</td>
<td>$k_d = \frac{1}{\pi} \left( \psi \cdot \pi + \sin(\psi \cdot \pi) \right)$</td>
</tr>
<tr>
<td></td>
<td>$k_q = \frac{1}{\pi} \left( \psi \cdot \pi - \sin(\psi \cdot \pi) + \frac{2}{3} \cos(\psi \cdot \pi) \right)$</td>
</tr>
<tr>
<td>PMSM. Surface magnets</td>
<td>$k_d = 1; \quad k_q = 1$</td>
</tr>
<tr>
<td>PMSM. Inset magnets</td>
<td>$k_d = \frac{1}{\pi} \left[ \psi \cdot \pi + \sin(\psi \cdot \pi) + \frac{1}{c_s} \left[ (\pi - \psi \cdot \pi - \sin(\psi \cdot \pi) \right) \right]$</td>
</tr>
<tr>
<td></td>
<td>$k_q = \frac{1}{\pi} \left[ \frac{1}{c_s} \left( \psi \cdot \pi - \sin(\psi \cdot \pi) \right) + \left( \pi - \psi \cdot \pi + \sin(\psi \cdot \pi) \right) \right]$</td>
</tr>
<tr>
<td>PMSM. Buried magnets</td>
<td>$k_d = \frac{4}{\pi} \left( \frac{\psi}{1 - \psi} \cos(\frac{\psi \cdot \pi}{2}) + \frac{\beta}{\tau_p} \right) \frac{D - 2 \cdot p \cdot h}{D}$</td>
</tr>
<tr>
<td></td>
<td>$k_q = \frac{1}{\pi} \left( \psi \cdot \pi - \sin(\psi \cdot \pi) \right)$</td>
</tr>
</tbody>
</table>

With: $\tau_p$ - pole pitch, $\psi$ - pole arc/ pole pitch, $h$ - permanent magnet height.

2. **Numerical determination of magnetizing inductance.**

The numerical determination of magnetizing inductance involves the realization of Finite Element Analysis (FEA) and the determination of the magnetic energy stored in the air gap. The following paragraphs describe the relations between this and the magnetic inductance. In addition we describe two ways for the calculation of stored energy; the first is by integration of density of energy and the second is by circuit modelling.

A. Magnetic energy stores in the air gap (uniform air gap machine).

If we consider an ideal machine with sinusoidal distribution of the induction along the air gap, that is,

$$B = \tilde{B} \cdot \sin \left( \frac{\pi \cdot x}{\tau_p} \right)$$

and we calculate the magnetic energy stored in the airgap, we obtain:

$$W = \mu_0 \cdot \frac{m^2}{2 \cdot \pi} \left( \frac{N \cdot q^2}{p} \right) \frac{D - L_m}{g_{eq}} \cdot I^2$$

If we combine (5) with (1) we can write:

$$W = \frac{m}{2} \cdot L_m \cdot I^2$$

usually $m = 3$. You can obtain the same expression if you consider the electrical circuit model with coupled coils.

For example for the induction machine model with 3 coils in the stator and 3 coils in the rotor, that is:

$$[L] = \begin{bmatrix} L_s & L_{rs} & L_{r} \end{bmatrix}$$

$$[L] = \begin{bmatrix} \frac{L_{a}}{3} & \frac{L_{ab}}{3} & \frac{L_{ac}}{3} \end{bmatrix}$$

$$[L] = \begin{bmatrix} \frac{L_{bar}}{3} & \frac{L_{bc}}{3} & \frac{L_{br}}{3} \end{bmatrix}$$

$$[L] = \begin{bmatrix} \frac{L_{abar}}{3} & \frac{L_{abc}}{3} & \frac{L_{abr}}{3} \end{bmatrix}$$

$$[L] = \begin{bmatrix} \frac{L_{bar}}{3} & \frac{L_{bc}}{3} & \frac{L_{br}}{3} \end{bmatrix}$$

$$W = \frac{1}{2} \sum L_{ij} \cdot I_i \cdot I_j$$

(7)

We obtain:

$$W = \frac{1}{2} \sum L_{ij} \cdot I_i \cdot I_j$$

(8)

(We omitted the terms with cos() to simplify the expression). If we consider the following values, corresponding to an instant with:

$$i_{a} = I; \quad i_{ba} = i_{bc} = -\frac{1}{2} I$$

$$i_{ar} = i_{br} = i_{cr} = 0$$

we obtain:

$$W = \frac{3}{2} L_m \cdot I^2 + \frac{3}{2} L_{cm} \cdot I^2$$

(10)

Except for the last term, this is the same expression (6). This term is a result to the dispersion effect and will be not considered for the main inductance calculation.
B. Magnetic energy stores in the salient pole machine.

We can obtain an expression for the magnetic energy stored in the case of the salient pole machine, but this takes longer to determine. We develop an expression based on circuit model approximation. In the salient pole machine we consider the first harmonic approximation for the inductance variation, i.e.,

\[ L \approx L_0 + L_2 \cos 2\theta_{cr} \] (11)

Figure 2. Salient pole machine

for the 3-phase synchronous machine we can write:

\[
\begin{align*}
L_{at} &= L_{at} + L_0 + L_2 \cos 2\theta_{cr} \\
L_{bt} &= L_{at} + L_0 + L_2 \cos (2\theta_{cr} + 2\pi / 3) \\
L_{ct} &= L_{ct} + L_0 + L_2 \cos (2\theta_{cr} - 2\pi / 3) \\
L_{at} &= -L_{at} / 2 + L_2 \cos 2\theta_{cr} \\
L_{bt} &= -L_{bt} / 2 + L_2 \cos (2\theta_{cr} + 2\pi / 3) \\
L_{ct} &= -L_{ct} / 2 + L_2 \cos (2\theta_{cr} - 2\pi / 3) \\
L_{df} &= L_{df} \cos \theta_{cr}, L_{df} = L_{df} \cos (\theta_{cr} - 2\pi / 3), \\
L_{df} &= L_{df} \cos (\theta_{cr} + 2\pi / 3)
\end{align*}
\] (12)

The energy stored is:

\[
W = \frac{1}{2} \left[ \frac{3}{2} N L_{at} \cdot i_a^2 + \frac{3}{2} N L_{at} \cdot i_b^2 + \frac{3}{2} N L_{at} \cdot i_c^2 + \frac{3}{2} N L_{ct} \cdot i_f^2 + \frac{3}{2} N L_{ct} \cdot i_f^2 \\
+ \frac{3}{2} N L_{df} \cdot i_d^2 + \frac{3}{2} N L_{df} \cdot i_f^2 \\n\right]
\] (13)

We omitted the terms with \( \cos() \) to simplify the expression. If we consider the following values, corresponding to an instant with:

\[
\begin{align*}
i_a &= I \\
i_b &= i_c = -I / 2 \\
i_f &= 0 \text{ (without field current)}
\end{align*}
\] (14)

We obtain the following expression:

\[
W = \frac{3}{2} I^2 \left[ \frac{3}{2} L_0 + \frac{3}{2} L_2 \cos (2\cdot \theta_{cr}) \right] + \frac{3}{2} L_{at} \cdot I^2
\] (15)

The last term is a result to the dispersion effect and will be not considered for the main inductance calculation.

If we consider two selected positions for the rotor, i.e.

- \( \cos(2 \cdot \theta_{cr}) = 1 \Rightarrow \text{Direct field orientation} \)
- \( \cos(2 \cdot \theta_{cr}) = -1 \Rightarrow \text{Quadrature field orientation} \)

Some after algebraic manipulations, we obtain:

\[
\begin{align*}
L_{rot} &= \frac{3}{2} \left( L_0 + L_2 \right) \\
W &= \frac{3}{2} I^2 \left( \frac{3}{2} L_0 + \frac{3}{2} L_2 \right) + \frac{3}{2} L_{at} \cdot I^2
\end{align*}
\] (16)

\[
\begin{align*}
L_{rot} &= \frac{3}{2} \left( L_0 - \frac{3}{2} L_2 \right) \\
W &= \frac{3}{2} I^2 \left( \frac{3}{2} L_0 - \frac{3}{2} L_2 \right) + \frac{3}{2} L_{at} \cdot I^2
\end{align*}
\] (17)

C. Inductance determination by means of flux concatenation

Another technique for the calculation of inductance is by the use of flux concatenation by a coil. If we consider a magnetic field distribution along the air gap, and its first harmonic, we can calculate the flux concatenation and the main inductance:

\[
L = \Phi \cdot \frac{1}{I} \cdot \frac{\int \nabla \times \vec{A} \cdot d\vec{S}}{I} = N \cdot \frac{\int \vec{A} \cdot d\vec{l}}{I}
\] (18)

If we consider a salient pole machine, we use a flux oriented over the direct and quadrature axis respectively, for the determination of direct and quadrature inductances.
3. Practical Applications

The following paragraphs show three examples of determination of main inductance. Two of them are compared with experimental results.

A. Asynchronous machine: 1.5 kW; 50 Hz; 220 / 380 V; 6.4 / 3.7 A; \( \cos \varphi = 0.85 \); \( 1420 \text{ min}^{-1} \); F class; \( J = 0.0105 \text{ kgm}^2 \); \( \Delta \). Connection.

Geometric and electrical data: 36/28 slots; 44 conductors/slot; \( D = 80 \text{ mm} \); \( g = 0.375 \text{ mm} \); \( L = 100 \text{ mm} \). We considered that \( k_c \cdot k_{sa} = 1.3 \) and \( \xi = 0.955 \).

![Figure 3. FEM model for asynchronous machine. Only ¼ of the machine has been modeled.](image)

<table>
<thead>
<tr>
<th>Method</th>
<th>( L_m ) (H)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical calculation</td>
<td>0.310</td>
</tr>
<tr>
<td>FEA</td>
<td>0.313</td>
</tr>
<tr>
<td>Experimental results</td>
<td>0.255</td>
</tr>
</tbody>
</table>

Table II. Main inductance for asynchronous machine.

B. Synchronous machine: 6 kVA; 220 V; 15.8 A; 50 Hz; 1500 min\(^{-1}\); Y connexion.

Geometric and electrical data: salient pole with uniform airgap (under the pole) \( g = 2 \text{ mm} \); \( D = 304 \text{ mm} \); \( L = 100 \text{ mm} \); \( \psi = 0.55 \); 36 slots; double layer lap winding; 5 conductors per slot and layer. We considered that \( k_c \cdot k_{sa} = 1.3 \) and \( \xi = 0.955 \).

![Figure 4. FEM model for synchronous machine. Direct field orientation](image)

<table>
<thead>
<tr>
<th>Method</th>
<th>( L_d ) (mH)</th>
<th>( L_q ) (mH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical calculation</td>
<td>9.84</td>
<td>4.25</td>
</tr>
<tr>
<td>FEA</td>
<td>10.7</td>
<td>4.23</td>
</tr>
<tr>
<td>Experimental results (reduced slip test)</td>
<td>7.42</td>
<td>5.30</td>
</tr>
</tbody>
</table>

Table III. Main inductance for synchronous machine.

C. Synchronous machine with permanent magnets. 5.1 Nm; 3500 min\(^{-1}\); \( I_N = 2.56 \text{ A} \); F class

Geometrical data: \( D = 80 \text{ mm} \); \( L = 68.9 \text{ mm} \); 36 slots; 6 pole; 35 conductors per slot; single layer lap winding; \( \zeta = 0.96 \); permanent magnet height \( h = 3 \text{ mm} \); \( g = 0.5 \text{ mm} \); \( \psi = 0.65 \); \( k_c \cdot k_{sa} = 1.3 \); surface permanent magnet.

In this case to impose \( i_f = 0 \) we change the PM characteristic from a non-magnetic material with the same magnetic permeability of the PM. This machine is considered as uniform air gap machine due to the value of recoil permeability of the PM (near to 1.0).

![Figure 5. FEM model for synchronous machine. Quadrature field orientation](image)

![Figure 6. FEM model for synchronous machine. Direct field orientation](image)
For this machine we determined the inductance by the method of flux concatenation. We obtain the magnetic field distribution and harmonic components showed in the figures 8 and 9.

The following table (IV) shows the calculated values.

<table>
<thead>
<tr>
<th>Method</th>
<th>L (mH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical calculation</td>
<td>6.55</td>
</tr>
<tr>
<td>FEA (energy)</td>
<td>5.15</td>
</tr>
<tr>
<td>Flux method (FEA)</td>
<td>6.0</td>
</tr>
</tbody>
</table>

### 3. Conclusions.

- We explained some methods to determine the main inductances for alternating current machines in an educational environment.
- We considered correction factors that are dependents on the pole-shape configuration.
- FEM is more precise than analytical calculation and is not dependent on an empirical or geometrical factors.
- Some of these experimental results are discordant with theoretical results due to estimation of some geometrical measures and magnetic characterization.

### Bibliography