Influence of Tidal Power Generation in the Self-Scheduling of a Price-Maker Producer

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Abstract.

In a pool-based electricity market, this paper addresses the self-scheduling problem faced by a price-maker owning some tidal power generation assets. The objective of this price-maker generator is to achieve maximum profit subject to market conditions and its own technical constraints. An exact and computationally efficient mixed-integer LP formulation of this problem is presented. This formulation models precisely both the price-maker capability of altering market-clearing prices to its own benefit and the particular operating conditions associated with tidal power production. Price-quota curves are used to describe the market and how the price-maker affects it. Two realistic case-studies are presented and its results are analyzed.

Keywords.

Price-maker, tidal power, electricity pool market, market power, mixed-integer programming.

1. Introduction

The objective of this paper is to study the influence of tidal power production when considering the optimal self-schedule for a price-maker producer[1] in an electric energy market. The self-scheduling problem of a price-maker producer is a problem that has been studied in different frameworks and for different market structures [1-3]. The main contribution of the present work is that it studies how the self-scheduling changes when one considers the possibility of tidal power production; obviously, the contribution of the tidal plant cannot be added ex-post and the problem has to be solved as one single optimization problem.

The framework considered is a pool-based electricity market for energy. In this market, producers send to the market operator production bids consisting on a set of energy blocks and their corresponding minimum selling prices for every of the 24 hours of the day-ahead market. Analogously, retailers send consumption bids that consist on a set of energy blocks and their corresponding maximum buying prices, also for every hour. In turn, the market operator clears the market using an auction algorithm and determines the hourly market-clearing prices and the accepted production and consumption energy bids. This hourly market-clearing price is paid to every producer and paid by every retailer in the corresponding hour. Both price-makers and competitive fringe producers are market agents. The load may be price-elastic or not. In the above context, this paper addresses the optimal self-scheduling problem of a price-maker producer who owns tidal power generation assets. As a price-maker, this generating company is capable of altering market-clearing prices.

All technical constraints conditioning the producer are precisely modeled through linear expressions.

[1] A price-maker producer is defined as a generating company that is capable of altering market-clearing prices, generally, to its own benefit.

The remaining of this paper is organized as follows. Section 2 provides the model that precisely considers all constraints related to the operation of tidal plants. In Section 3, the self-scheduling problem faced by the price-maker is precisely formulated. In Section 4, both models are merged in order to produce one model that takes completely into account all the operating conditions of the price-maker under consideration. In Section 5 two realistic simulations are analyzed and their results are compared. Finally, Section 6, provides some conclusions.

2. Tidal Power Modeling

A simple description of the formulation needed to model all the features of tidal power plants is presented next. The author of this paper presented in [4] a full model for the operation of this type of plant.

2.1. Introduction to tidal Power

Tidal plants are built to extract energy from the tidal cycles that take place during the day at certain coastal locations. Around the world these places are quite a few, most significantly the Bay of Fundy in Canada and La Rance estuary in France. Tidal power is an energy source that produces no contamination and consumes no fuel. However, tidal power requires the building of large dams and has a significant impact on seashores and marine wildlife. Tidal power constitutes a relevant source of electrical energy for these places where tidal ranges are large and where environmental impact can be controlled.

References [5-7] provide a worldwide overview of either tidal power facilities or potential sites. The work presented in this paper is related with the pioneering work of Dechamps and others, carried out in the late seventies and early eighties to evaluate the installation of the tidal power facility in the Bay of Fundy, in Canada, [8,9]. Background on tidal power plants can also be found in the following references: [5-7,10-12].

2.2. Operating Cycles for Tidal Power Plants

To plan the operation of a tidal power plant many technical and economic aspects have to be considered. Basically, one has to take into account the market prices for the time horizon and the changing level of the sea.

Tides have a cycling nature, hence it is reasonable to operate tidal plants in a cycling manner. An “operating cycle” is defined as the set of actions taken by a plant operator during a tidal cycle. In the following paragraphs the most important operating cycles are presented.

Single Effect Cycle

The single effect cycle is the simplest way to operate a tidal power plant. It implies power production in only one flow direction. Two versions of this operating cycle are possible depending on the flow direction that is used to produce power: either power is generated as water flows out of the reservoir, (ebb generating cycle), or as water flows in the reservoir (flood generating cycle). For different reasons, "ebb generating cycles" are preferred.

The operation on a single effect cycle is as follows: when the tide reaches its maximum, the reservoir is full and the sluices connecting it to the sea are closed. After some hours, the sea level is lower than the reservoir level because the sluices have remained closed; then, the generator is started and power is produced, this is possible thanks to level difference between reservoir and sea. This functioning can only be maintained for a small number of hours, because the sea level eventually reaches its minimum and goes up again and because the reservoir level decreases as water flows through the turbine to produce power. Eventually, both levels equal and no more power can be produced. In this moment, sluices connecting the sea and the reservoir are opened so that the reservoir fills again with water from the sea. Finally, the maximum tide is reached and the process is repeated.

If pumping is available to increase the difference in levels between the reservoir and the sea, it must take place before starting power production. Note that water pumping is not very costly because difference in height between the reservoir and the sea is not high. However, some hours later, when the same water flows back to the sea, the difference in height will be larger, and hence, the cost of pumping is more than recovered.

Double Effect Cycle

A simple way to improve the performance of the single effect cycle is to allow the generator to produce power in both directions[2]. The only difference is that when water level is higher in the sea than in the reservoir, instead of simply admitting water in the reservoir, the plant is operated to produce energy again.

Multiple Effect Cycle

The last type of cycle commonly described in the literature is the multiple effect cycle. This cycle allows a tidal plant to produce at a constant level of power during the whole time horizon, improving efficiency and avoiding start-ups and shut-downs. However, this cycle requires building an extra dam to divide the reservoir in two. This extra investment is a significant economic barrier to the implementation of multiple effect cycles.

Figure 1 illustrates the similarities and differences between the three cycles described above.

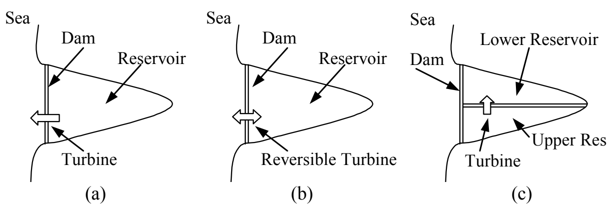

Figure 1. Different operating cycles: (a) Single effect cycle. (b) Double effect cycle. (c) Multiple effect cycle.

This paper concentrates on the double effect cycle, which is the most attractive from an economical viewpoint. The paper provides a detailed mathematical model for this cycle, which can be straightforwardly adapted to the other two previously defined cycles.

[2] The simplest way to do this is to install a reversible machine that is able to generate regardless the direction of the water flow.

2.3. Modeling Tides

The first aspect that has to be taken into account to analyze the functioning of a tidal power plant is the tide itself. Tides take place twice a day, and are due to the attraction forces that mainly the moon, but also the sun, impose on water bodies all over the earth. Recorded data in tabular form can be used to model the evolution of water level. Once tide height data is available, the difference in levels between the sea and any reservoir, x_t, can be computed as:

$$x_t = h_t^{res} - h_t \qquad \forall t \in \Omega_T \qquad (1)$$

where h_t^{res} is the water level in the reservoir, h_t is the tide height. Variable x_t is fundamental in the modeling of the power produced by a turbine (and of the power consumed if the machine is pumping).

Note that different days have different tides, and hence, a multi-column table is needed if multi-day studies are made. Figure 2 shows a real example of the evolution of tide height for Portland, Maine [13]. The two different tides that take place everyday can be observed in this figure.

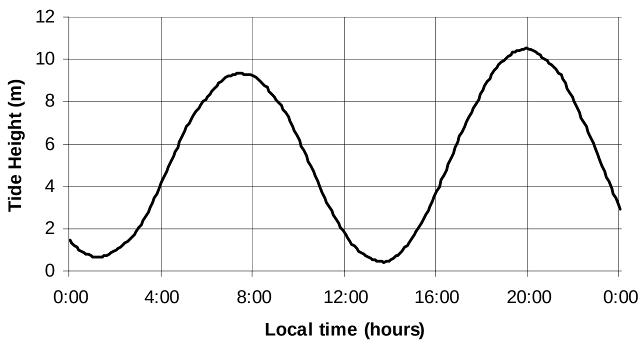

Figure 2. Evolution of tide in Portland, Maine, in July 10, 2003.

2.4. Volume-Height Relationship in Reservoirs

An important aspect of hydro generation, and particularly in tidal generation, is the relationship that exists between the reservoir volume and the height reached by the water at the dam barrage. That is,

$$v_t = g(h_t^{res}) \quad \text{or} \quad h_t^{res} = g^{-1}(v_t) \qquad \forall t \in \Omega_T \qquad (2)$$

where v_t is the total water volume contained in the reservoir, as in (2), and h_t^{res} is as in (1). In general, (2) is a nonlinear and complex relationship. Linear models are too rough a simplification, because for these models the relation between height and volume implies a reservoir of constant section, which is not realistic. Piece-wise linear models provide an appropriate approximation. A piece-wise linear model can be implemented through the following set of linear constraints, which includes binary variables:

$$v_t = \sum_{g \in \Omega_G} \alpha_g \cdot n_{tg} \qquad \forall t \in \Omega_T \qquad (3)$$

$$h_t^{res} = \sum_{g \in \Omega_G} n_{tg} \qquad \forall t \in \Omega_T \qquad (4)$$

$$n_{tg} \le y_{t,g-1} \cdot N \qquad \forall g > 1, g \in \Omega_G, \forall t \in \Omega_T \qquad (5)$$

$$n_{t,1} \le N \qquad \forall t \in \Omega_T \qquad (6)$$

$$n_{tg} \geq y_{tg} \cdot N \qquad \forall g \in \Omega_G, \forall t \in \Omega_T \qquad (7)$$

$$y_{tg} \in \{0,1\} \qquad \forall g \in \Omega_G, \forall t \in \Omega_T \qquad (8)$$

where Ω_G is the set of steps used for the piece-wise linear approximation of (2) that contains G blocks, α_g is the slope of the g-th step of the linearized function that relates volume and height, n_{tg} is a variable defined for every step that expresses the part of the step that is needed to obtain a total height of h_t^{res}, N is the maximum value for n_{tg}, and y_{tg} is a binary variable that is equal to 1 if and only if the k-th step is completely used to achieve a total height of h_t^{res}. Equation (3) provides the piece-wise approximation for the volume and equation (4) the piece-wise approximation for the height. Constraints (5)-(7) enforce the logic of the approximation through binary variable y_{tg}. Figure 3 illustrates the formulation (3)-(8) presented above.

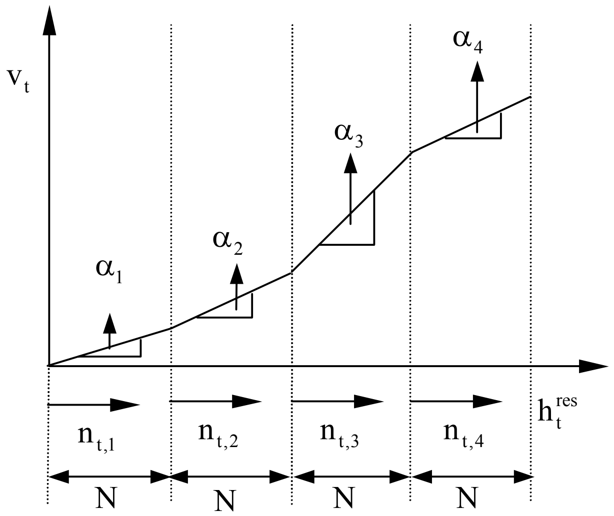

Figure 3. Piece-wise approximation of the relationship between the reservoir volume and its height. Four blocks are used.

2.5. Water Balance

Equations stating the mass conservation principle are needed to formulate hydro problems. These equations state that the increment in the amount of water in any reservoir equals the amount of water that entered the reservoir minus the amount of water that left it. Water balance equations are now presented for both single effect cycle and double effect cycle.

Water Balance. Single Effect Cycle.

The following equation states water balance for single effect cycles:

$$v_t - v_{t-1} = \varepsilon \cdot [b_t - u_t + s_t] \ \forall t \in \Omega_T \qquad (9)$$

where ε is the length of the time period considered, usually 1 hour; b_t is the flow of water from the sea into the reservoir due to pumping during time period t; u_t is the flow of water from the reservoir to the sea during time period t to generate electric energy and s_t is the amount of water flowing from the sea into the reservoir during time period t without generating electric energy; this is needed in single effect cycles to refill the reservoir once it has been emptied; finally, v_t is the volume of the reservoir in time period t.

Water Balance. Double Effect Cycle

The difference between single and double effect cycles is that in double effect cycles water can be used to generate energy in both directions and can be pumped in both directions. The equation stating water balance for double effect cycles is also (9), but its interpretation is a little bit different. In double effect cycles, both b_t and u_t in (9) can be either positive or negative. A positive value for u_t means that, in time period t, energy is generated by water flowing from the reservoir to the sea (ebb generation). If u_t is negative, power is generated by water flowing from the sea into the reservoir (flood generation). Similarly, b_t is positive for pumping during an ebb period and negative for pumping during flooding periods. Finally, s_t is positive for water admissions during ebb periods and negative for water discharges during flooding periods.

2.6. Power Production Models. Linear Approximations

In the most general case, power produced by a turbine can be regarded as a function of two variables, height and flow, which is expressed as:

$$p_t = f(x_t, u_t) \qquad \forall t \in \Omega_T \qquad (10)$$

where u_t is the amount of water that flows through the turbine, as in (9). In general, expression (10) is complicated and needs to be approximated. So called “Hill chart” curves are useful approximations. The Hill chart provides a set of curves relating power produced with water flow, being each curve associated to a different value of height [14]. Figure 4 shows an example of convex Hill chart curves.

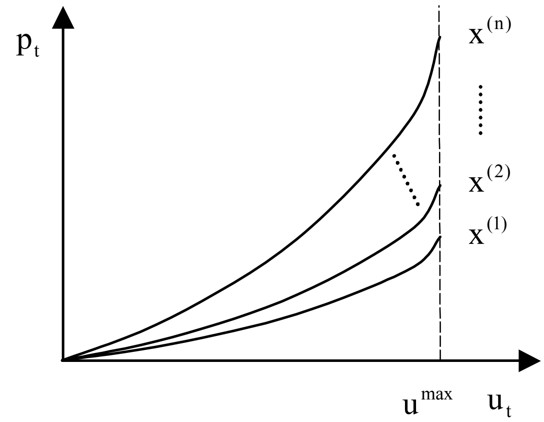

Figure 4. Hill chart. Production curves for different values of the difference in height between the reservoir and the sea.

In Figure 4 three different curves are presented, each of them is considered valid for a range of height difference between the reservoir and the sea. In that figure, note that $x^{(1)} < x^{(2)} < ... < x^{(n)}$. Note also that the higher the height, the higher the power produced.

The formulation presented in this paper uses a convenient piece-wise linear approximation of the curves in the Hill chart. This is done by means of binary variables, and results in a mixed-integer linear formulation. The equations needed are presented below.

The range of possible values for x_t is defined as:

$$-x^{max} \le x_t \le x^{max} \qquad \forall t \in \Omega_t \qquad (11)$$

Note that x_t is positive if the reservoir level is higher than the sea level, and negative if the reservoir level is lower than the sea level. For single effect ebb generating cycles and for multiple cycles, only the positive part of the range defined in (11) is of practical interest; because power cannot be generated or consumed for negative values of x_t. However, for double effect cycles, energy generation and pumping are possible in both directions, hence, the whole range defined in (11) is of interest.

Hill chart curves contain a large number of curves for different values of the absolute value of x_t: $|x_t|$. Two new variables are required to model absolute value functions: x_t^+ and x_t^-. The following equations relate the values of the new variables to the value of the original variable x_t and define the limits for the new variables:

$$x_t = x_t^+ - x_t^- \qquad \forall t \in \Omega_t \qquad (12)$$

$$|x_t| = x_t^+ + x_t^- \qquad \forall t \in \Omega_t \qquad (13)$$

$$0 \le x_t^+ \le x^{max} \cdot w_t \qquad \forall t \in \Omega_t \qquad (14)$$

$$0 \le x_t^- \le x^{max} \cdot (1 - w_t) \qquad \forall t \in \Omega_t \qquad (15)$$

$$w_t \in \{0,1\} \qquad \forall t \in \Omega_T \qquad (16)$$

Equation (13) is presented for clarity, however, this non-linear relation is not imposed as a constraint in the models. Note that from (14)-(16), either x_t^+ or x_t^- has to equal 0 for every time period; in fact, if binary variable w_t is 0, only x_t^- can be positive, and if w_t is 1, only x_t^+ can be positive. Binary variable w_t is 1 if the level in the reservoir is higher than the sea level ($x_t > 0$) and 0 if the level in the reservoir is lower than the sea level ($x_t < 0$). Binary variables w_t are required to formulate important constraints presented below in the paper. Note that equation (11) can be rewritten as:

$$|x_t| \le x^{max} \qquad \forall t \in \Omega_t \qquad (17)$$

To make the computational model tractable, a reduced number of curves from the Hill chart are used. The range defined in (17) can be divided into H+1 different subregions by selecting H different intermediate height points inside that range. In particular, to make all subregions equal, the H different points have to be chosen as:

$$L_h = \frac{h}{H+1} x^{max} \quad h \in \Omega_H, \forall t \in \Omega_T \qquad (18)$$

where Ω_H is a set that contains all the subregions defined over (17). Once the subregions have been defined, one of the curves from the Hill chart has to be selected to represent the relationship between power produced and water flow for values of height $|x_t|$ inside that subregion, in which height is assumed constant.

A set of binary variables d_{th} is defined so that their values depend on the height subregion in which the actual value of $|x_t|$ lies for every time period. The value for binary variable d_{th} is 1 if and only if the upper limit of height subregion h is lower than the actual value of $|x_t|$. The formulation is as follows:

$$|x_t| \ge \sum_{h \in \Omega_H} d_{th} \cdot L_h - \sum_{h \in \Omega_H, h>1} d_{th} \cdot L_{h-1} \quad \forall t \in \Omega_T \qquad (19)$$

$$|x_t| \le L_1 - \sum_{h \in \Omega_H} d_{th} \cdot L_h + \sum_{h \in \Omega_H, h>1} d_{t,h-1} \cdot L_h \quad \forall t \in \Omega_T \qquad (20)$$

$$d_{th} \ge d_{t,h+1} \qquad \forall t \in \Omega_T, \forall h \in \Omega_H \qquad (21)$$

$$d_{th} \in \{0,1\} \qquad \forall t \in \Omega_T, \forall h \in \Omega_H \qquad (22)$$

Note that, if $|x_t|$ is between 0 and L_1, all binary variables should be 0; if $|x_t|$ is greater than L_1, but lower than and L_2, then, the first binary variable is 1 and the rest are 0, and so on.

Equations (19)-(22) allow the correct selection of the values of the binary variables d_{th} for every height subregion of region (17). In particular, equations (19) and (20) define the lower and upper limits for variable $|x_t|$, respectively; equation (21) states that a variable can only be 1 if all previous variables are 1 already, thus forcing the correct sequence of values for the binary variables.

Hill charts are defined only for positive values of the water flow, u_t. However, as previously stated, double effect cycles may produce energy from flows of water from either direction. This is the reason that forces the formulation of production curves in terms of the absolute value of the water flow. The same considerations can be made on the water flow due to pumping, b_t. Four new variables are defined to formulate the absolute values of both u_t and b_t. The following equations relate the new variables to the original variables:

$$u_t = u_t^+ - u_t^-; \quad |u_t| = u_t^+ + u_t^- \qquad \forall t \in \Omega_t \qquad (23)$$

$$b_t = b_t^+ - b_t^-; \quad |b_t| = b_t^+ + b_t^- \qquad \forall t \in \Omega_t \qquad (24)$$

Any curve defined through the Hill chart is then approximated using a piece-wise linear approximation through equations (25)-(33) below:

$$p_t - \sum_{\ell \in \Omega_L} \left(\left(u_{t\ell}^+ + u_{t\ell}^- \right) \cdot \rho_{h\ell} \right) - p^{max} \cdot (h - 1 - B_{th}) \le 0 \qquad (25)$$
$$\forall t \in \Omega_T, \forall h \in \Omega_H$$

$$p_t - \sum_{\ell \in \Omega_L} \left(\left(u_{t\ell}^+ + u_{t\ell}^- \right) \cdot \rho_{h\ell} \right) + p^{max} \cdot (h - 1 - B_{th}) \ge 0 \qquad (26)$$
$$\forall t \in \Omega_T, \forall h \in \Omega_H$$

$$B_{th} = \sum_{k \in \Omega_H, k \ge h} d_{tk} - \sum_{k \in \Omega_H, k<h} d_{tk} \qquad \forall t \in \Omega_T, \forall h \in \Omega_H \qquad (27)$$

$$u_t^+ = \sum_{\ell \in \Omega_L} u_{t\ell}^+ \qquad \forall t \in \Omega_T \qquad (28)$$

$$0 \le u^{+}_{t\ell} \le u^{max}_{\ell} \qquad \forall t \in \Omega_T, \forall \ell \in \Omega_L \qquad (29)$$

$$u^{-}_{t} = \sum_{\ell \in \Omega_L} u^{-}_{t\ell} \qquad \forall t \in \Omega_T \qquad (30)$$

$$0 \le u^{-}_{t\ell} \le u^{max}_{\ell} \qquad \forall t \in \Omega_T, \forall \ell \in \Omega_L \qquad (31)$$

$$u^{+}_{t} \le u^{max} \cdot w_t \qquad \forall t \in \Omega_T \qquad (32)$$

$$u^{-}_{t} \le u^{max} \cdot (1 - w_t) \qquad \forall t \in \Omega_T \qquad (33)$$

where, p^{max} is the maximum power that the plant is able to produce, and u^{max} is the maximum water flow admitted through the turbine. Note that, then, equations (25)-(26) are not binding unless the term $(h-1-B_{th})$ is equal to 0. Equation (31) is formulated in such a way that the term $(h-1-B_{th})$ is equal to 0 only for the subregion in which the correct value of $|x_t|$ lies. For the other subregions, equation (27) implies that equations (25)-(26) are not binding, i.e, $(h-1-B_{th}) > 0$. If equations (25)-(26) become binding, they state that p_t has to equal the piece-wise linear approximation related to the corresponding subregion (note that the second terms in (25) and (26) are identical). Mathematically, if $(h-1-B_{th})$ is equal to 0, and assuming $w_t = 1$ ($x_t > 0$), combining (25) and (26) results in the following constraint:

$$p_t = \sum_{\ell \in \Omega_L} \left(u^{+}_{t\ell} \cdot \rho_{h\ell} \right) \qquad (34)$$

Note that (34) is the expression of the piece-wise linearization of the power produced for a given subregion of the Hill chart. The summation is made over the different blocks used to linearize power production. Parameter $\rho_{h\ell}$ is the slope of the linearized production curve for the ℓ-th block of the linearization and for the h-th height subregion. Note that depending on the subregion considered, a different set of values for $\rho_{h\ell}$ is used. Within the corresponding subregion, (34) states that p_t does not depend of x_t; this dependence is indirectly considered through the use of different height subregions.

Figure 5 illustrates equation (34). This figure depicts a convex piece-wise linear approximation of the production curve associated to a height subregion. If a nonconvex approximation is required, it can be achieved using additional binary variables.

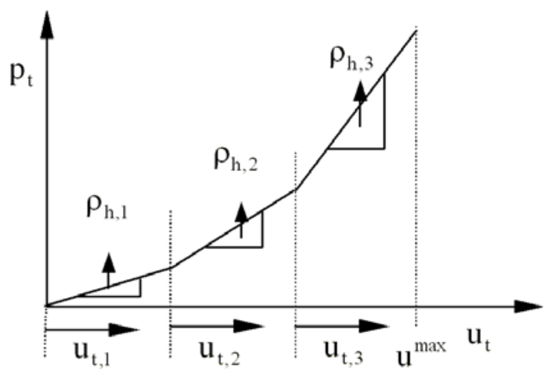

Figure 5. Piece-wise linear approximation of one of the production curves from the Hill chart. Three blocks are used.

Equations (28)-(33) define variables u^{+}_{t} and u^{-}_{t}.

The same considerations made to obtain a formulation of p_t based on the value of $|x_t|$ have to be made to obtain a formulation related to the operation of the plant in pumping mode. Note that the only relevant difference would be the introduction of an efficiency term μ to consider the fact that water pumping should be less effective than power production; otherwise, a pumping-production cycle will be costless, which is unrealistic. For details on this formulation, see [4].

The formulation presented above is valid for double effect cycles. For single effect cycles and for multiple effect cycles, some simplifications have to be made to properly consider the fact that water can be pumped in only one direction and energy can be produced by water flowing in only one direction. This simplifications can be made by fixing a value of zero for both u^{-}_{t} and b^{-}_{t}.

2.7. Other Constraints

Additional constraints are needed to enforce the logic of operation of tidal power plants. This constraints are presented below, their mathematical formulation is presented in [4].

- Pumping and energy generation cannot take place at the same time.
- Free entrance of water is only possible if the value of x_t allows it.
- A final constraint is needed to ensure that the initial level in the reservoir equals its final level.

3. Market Modeling

As previously stated, this paper addresses the self-scheduling problem of a price-maker. The objective function of this self-scheduling problem is to maximize the price-maker profits. Once the optimal self-schedule is known, an appropriate bidding strategy to actually achieve this optimal schedule should be developed.

For every hour, it is assumed that the market-clearing price as well as the offer and demand curves are available once the market has been cleared. This is the case of several electricity markets like the market in mainland Spain [15], the former electricity market of California [16] and the electricity market of New England [17]. The above information is crucial because it allows small producers to forecast next-day market-clearing prices, and it also allows price-makers to forecast their corresponding price-quota curves. Note that several price-makers can compete in the considered pool-based electricity market.

Background references on electricity markets are [18-20]. Self-scheduling references for thermal producers include [1, 21], and for hydro producers [14, 22-23].

3.1. Price-quota curve description

For a given hour, the quota of a price-maker is the amount of power it contributes to serve the demand in that hour. If the price-maker exercises its market power by retaining production, the market-clearing price increases. The curve that expresses how the market-clearing price changes as the quota of the price-maker changes is called residual demand curve [20] or, more directly, price-quota curve. The price-quota curve for a given hour, corresponding to a price-maker, is generally stepwise monotonically decreasing. Note that different price-makers competing in the same electricity market present different price-quota curves. As an example, Figure 6 shows a typical price-quota curve.

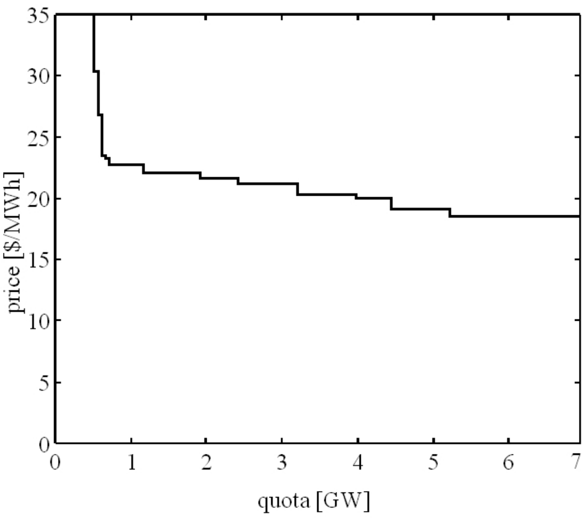

Figure 6. Price-quota curve

The 24 hourly day-ahead price-quota curves of a given price-maker provide all the market information it needs to self-schedule optimally, i.e. to maximize its benefits. That is, these curves comprise the effects of all interactions with competitors and the market functioning rules. Once these curves are available, the price-maker self-scheduling problem can be precisely formulated, independently of the problems of other producers.

In a day-ahead electricity market each producer uses a self-scheduling algorithm to determine its optimal self-schedule. Then, each producer uses a bidding strategy to achieve in the market that optimal schedule. Finally, the market operator uses a market clearing procedure to determine the actual production for each producer.

The day-ahead price-quota curves of a price-maker can be obtained (i) by market simulation or (ii) using forecasting procedures [20]. In this paper the price-quota curves of the price-maker are assumed to be known data.

3.2. Nonlinear formulation

The natural formulation of the optimization problem that a price-maker has to face is nonlinear, due to the products between the variables that appear in the objective function [1].

This formulation is:

$$\underset{p_{t,i},q_t}{\text{maximizar}} \quad \sum_{t=1}^{T}\left[\lambda_t(q_t)q_t - \sum_{i=1}^{m} c_{t,i}\right] \tag{35}$$

$$\text{sujeto a}: \quad p_{t,i} \in \Pi_i; \quad i=1,...,m \quad t=1,...,T; \tag{36}$$

$$q_t = \sum_{i=1}^{m} p_{t,i}; \qquad t=1,...,T \tag{37}$$

The objective function (35) expresses the profit of the price-maker over the planning horizon. The first term is the total revenue and the second one is the total production cost, as formulated in [1,21]. Set of constraints (36) enforces that every unit works within its feasible operating region over the whole planning horizon. A precise mixed-integer linear description of this feasibility region can be found in [21,24]. Set of constraints (37) expresses for every hour the price-maker quota as the sum of the power production of its units.

3.3. Linear formulation

An alternative equivalent formulation of problem (35)-(37) that is linear is provided below:

$$\underset{p_{t,i},q_t,b_{t,s},u_{t,s}}{\text{maximize}} \quad \sum_{t=1}^{T}\left[\sum_{s=1}^{n_t}\lambda_{t,s}(b_{t,s}+u_{t,s}q_{t,s}^{min}) - \sum_{i=1}^{m} c_{t,i}\right] \tag{38}$$

$$\text{subject to} \quad p_{t,i} \in \Pi_i; \quad i=1,...,m; \; t=1,...,T \tag{39}$$

$$q_t = \sum_{i=1}^{m} p_{t,i}; \qquad t=1,...,T \tag{40}$$

$$q_t = \sum_{s=1}^{n_t}(b_{t,s}+u_{t,s}q_{t,s}^{min}); \quad t=1,...,T \tag{41}$$

$$0 \le b_{t,s} \le u_{t,s}b_{t,s}^{max}; \quad s=1,...,n_t; \; t=1,...,T \tag{42}$$

$$\sum_{s=1}^{n_t} u_{t,s} = 1; \qquad t=1,...,T \tag{43}$$

The objective function (38) expresses the profit of the price-maker: total revenue minus total costs. Taking advantage of the stepwise nature of the price-quota curve in every hour, the total revenue is expressed linearly using real variables $b_{t,s}$ and binary variables $u_{t,s}$. Quota values that originate discontinuities on revenues do not lead to an ambiguous formulation, because the maximization of the objective function always leads to the highest revenue values. Sets of constraints (39) and (40) are identical to (36) and (37). Set of constraints (41) expresses linearly the price-maker quota in every hour as a function of variables $b_{t,s}$ and $u_{t,s}$. Block of equations (42) expresses that the blocks of the price-quota curve of every hour are nonnegative values, bounded above. Block of equations (43) states that only one variable $u_{t,s}$ is different from 0 in every hour. Thus, sets of equations (42) and (43) together enforce that only one variable $b_{t,s}$ is different from 0 in every hour. It should be noted that both formulations (35)-(37) and (38)-(43) are fully equivalent, and this allows saying that the linear formulation (38)-(43) is exact.

4. Problem Formulation

Considering the models presented in parts 2 and 3 of this paper, a full model can be constructed that includes all constraints and adequately takes into account the particular features of both tidal power generation and thermal generation to obtain the optimal self-schedule in the market. Particularly, this is a mixed-integer linear optimization problem, and can be solved by means of commercially available software.

5. Case Studies

The considered electricity market includes one price-maker producer owning 40 thermal units and one tidal energy plant. The market also includes competitive fringe producers comprising a total 120 thermal units. It should be noted that mixed hydro-thermal price-makers can be analyzed in a similar way as thermal price-makers. The market time horizon is 24 hours. For the case-studies, the optimization problems are solved using CPLEX 7.0 under GAMS [25]. Data for all units are based on the 1996 IEEE RTS [26], and are detailed in Table I. In this table, 'Type' indicates the unit type (A, B, C, D, E, F or G); 'PM/CF' indicates the number of units corresponding to the price-maker and the competitive fringe producers, respectively; $\overline{P}$ and $\underline{P}$ indicate respectively maximum and minimum power output; every 'C_j' value provides the production cost of the block 'j' of the unit (four-block piecewise convex cost curves are considered); 'RR' gives both ramp-up and ramp-down maximum values; and 'SC' is the constant start-up cost. Note that minimum up and down time constraints are also enforced.

Table I. Generating units data

Type	A	B	C	D	E	F	G
PM / CF	6 / 18	6 / 18	6 / 18	6 / 18	6 / 16	6 / 16	4 / 16
$\overline{P}$	12	76	100	155	197	350	400
$\underline{P}$	2.4	15.2	25	54.25	68.95	140	100
$C_1^{(*)}$	23.41	11.46	18.60	9.92	19.20	10.08	5.31
$C_2^{(*)}$	23.76	11.96	20.03	10.25	20.32	10.68	5.38
$C_3^{(*)}$	26.84	13.89	21.67	10.68	21.22	11.09	5.53
$C_4^{(*)}$	30.40	15.97	22.72	11.26	22.13	11.72	5.66
RR[MW/h]	12	76	100	155	180	120	400
SC($)	196	1353	1635	2173	2239	10190	NA(**)
(*) Units: [$/MWh]. (**) NA: Not Applicable.							

The tidal energy plant owned by the price maker has a maximum power output of 500 MW and maximum water flow of 400 m^3/s; pumping is allowed whenever necessary; tide amplitude at the plant's location is 3 meters, i.e., the difference between the maximum tide height and the average tide height is 3 meters, being the maximum range (maximum minus minimum tide height) twice the amplitude: 6 meters.

Parameter ρ from the power-height-flow relationship in the plant is presented in Table II; note that only one value of parameter ρ is presented for each height range; this is because a single block has been used for the linearization of each of the production curves. Data regarding parameter α_g related to the volume-height relationship (see 27) is presented in Table III.

Table II. Values of ρ for the power-height-discharge curve

Height Range [m]	0 - 0.2	0.2 - 2.25	2.25 - 4.5	4.5 - 6.75	6.75 - 9.0
Value for ρ [$MW/(m^3/s)$]	0.000	0.001	0.010	0.020	0.030

Table III. Parameter α_g defining the reservoir.

Height Range [m]	0.0 - 2.8	2.8 - 5.6	5.6 - 8.4	8.4 - 11.3
Value for α_g [m^2]	300	600	900	1100

In this framework two different case-studies are performed. In the first case-study, the whole model is solved and profits are calculated for the price-maker. In a second case-study, tidal power production is not included in the model. Comparison is made between profits earned by the price-maker for both case studies.

In order to lower the computational load of the problem, one simplification was introduced to the formulation of the tidal part of the problem: only odd hours where considered in the formulation; results for even hours were calculated by extrapolation from previous hours. This simplification reduces to 12 the number of hours considered in the tidal part of the problem, this 12 hours do represent two full tide cycles for the plant. This simplification makes the problem tractable, otherwise, no solution could be found within a reasonable time, see computing times in Table IV below.

Results from Table IV show that the reduction in profits due to not including tidal power generation is approximately 3.5%, which, for real-life companies implies plenty of money.

Table IV. Some results from the case-studies

	Case 1: Base Case	Case 2: No Tidal Plant
Company Profits [M$]	899,24	869,12
Computing time [s]	8421,8	87,63
Net Tidal Production [GWh]	2,2	-
Total Production [GWh]	95,053	96,174
Average Price [$/MWh]	18,138	18,138

The resulting market-clearing prices for the first case-study are presented in Figure 7.

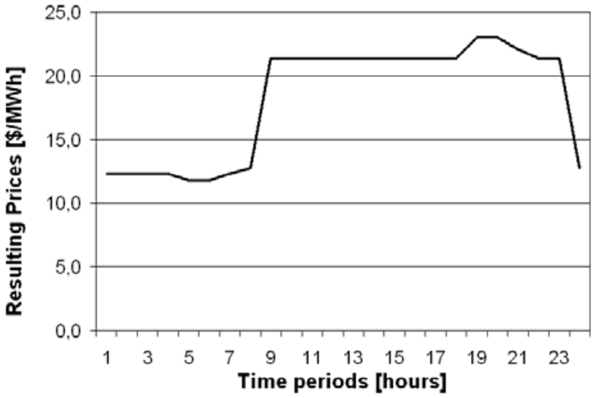

Figure 7. Resulting Market-Clearing Prices

The results for the tidal plant in the first case-study are presented in Figures 8 and 9; the three plots presented in these figures provide, for every hour: water level inside the reservoir, sea level and power produced by the plant,.

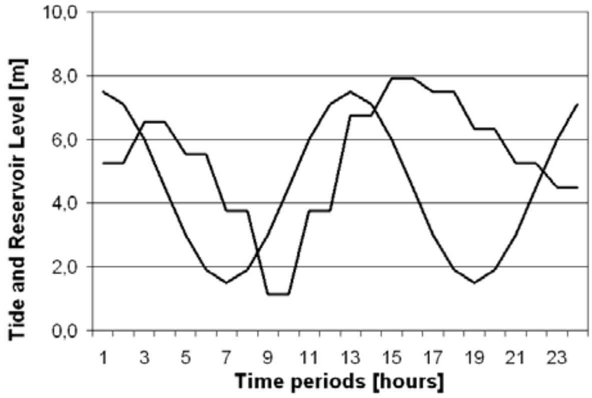

Figure 8. Tide height and reservoir level for all hours

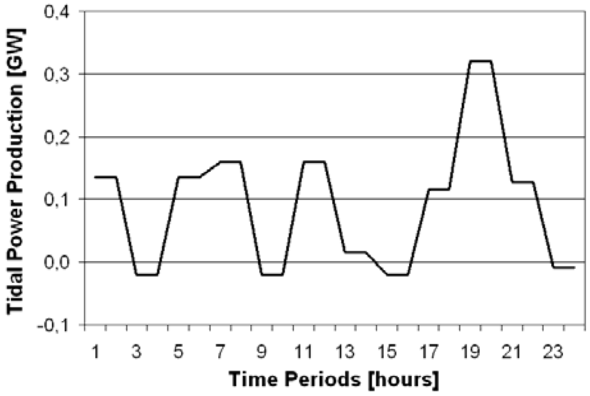

Figure 9 Power production for all hours

Note that in Figure 9, the negative values for power production have to be understood as water pumping. Also, note from Figure 9 that all hours are "active hours" in the sense that the machine is used; when the difference in height between the reservoir and the sea is high enough, power is produced; when it is not enough, like during hour 3 (see Figure 8), then, the machine is set to pump water, hence increasing the amount of water that will be used for power production later. Also note that the biggest amount of power production takes place during the hours with the highest prices (compare Figure 7 with Figure 9). Note that this behavior is similar in many ways to the behavior of pumped-storage facilities used by some power producers; i.e., water pumping during low price hours allows for power production during price peaking hours.

6. Conclusions

This paper provides a mixed-integer LP formulation for the self-scheduling problem faced by a price-maker who owns some tidal power generation. This formulation is exact in the sense that it is equivalent to a 'natural' nonlinear formulation of the problem. This linear formulation allows an efficient solution using standard mathematical programming methods. The influence of the tidal power generation is clear from the case studies; profits can increase as much as 3.5% thanks to the use of tidal power generation; for real companies, this increase implies earning thousands of euros more per day.

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