Flow of molten metal in a pipe driven by electromagnetic field

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1. Introduction

Many industrial technologies working with molten metals (or other electrically conductive liquids such as acids or solutions of salts) are based on force and thermal effects of electromagnetic field. Typical are, for example, pumping, dosing or stirring and other similar processes.

Mathematical and computer modeling of these processes (carried out for the purpose of their optimization) is still a challenge. These tasks represent multiply coupled nonstationary and often nonlinear problems characterized by interaction of several physical fields (usually electromagnetic field, temperature field and field of flow) influencing one another. Even when high attention is paid to the topic (the books and papers in the domain abound, see, for example, [1-3]), a lot of tasks still remain unresolved.

The paper presents the solution of flow of molten metal in a ceramic pipe. The flow is driven by the Lorentz forces produced by harmonic magnetic field generated by the field coils appropriately arranged along the pipe. Checked is also the temperature rise of melt due to the Joule losses produced by eddy currents in it. The numerical solution is carried out (unlike former classical FEM and FVM-based algorithms) by our own SW HERMES based on the *hp*-FEM.

Key words: Induction heating, molten metal, *hp*-finite element method, electromagnetic field, temperature field, field of flow.

2. Formulation of the problem and its mathematical model

Consider incompressible electrically conductive liquid **1** in a horizontal ceramic pipe **2** in the form of a circular ring. The basic arrangement is depicted in Fig. 1. The field coils **3** placed on both sides of the channel carry harmonic currents of suitable phase shifts. These currents must produce in melt magnetic field with a significant radial component that contributes to the axial component of the Lorentz force. In order to suppress leakage of the magnetic flux outside the system, the whole arrangement can be inserted into an aluminum shielding shell **4**.

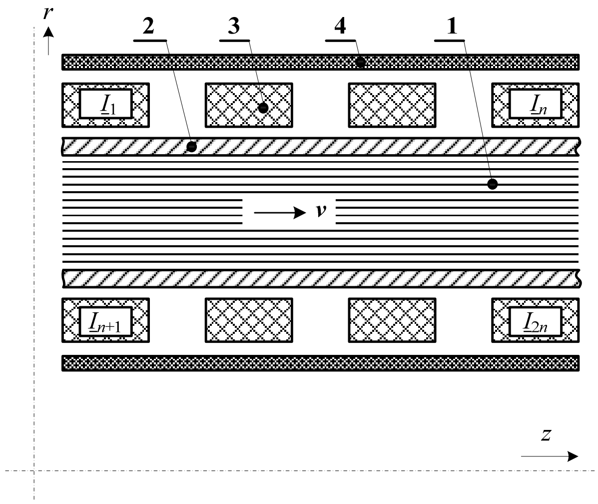

Fig.1: The investigated arrangement:
1 – electrically conductive liquid, **2** – ceramic pipe,
3 – field coils, **4** – shielding shell

The mathematical model of the problem consists of three PDEs describing the distribution of the involved physical fields. On the other hand, the arrangement may be considered 2D, which leads to significant simplifications for the numerical model.

Electromagnetic field in the system is harmonic and linear (its definition area is supposed to contain no ferromagnetic parts). In such a case its distribution is described in terms of the phasor of magnetic vector potential $\underline{\boldsymbol{A}}$ by equation [4]

$$\mathrm{curl}\,\mathrm{curl}\,\underline{\boldsymbol{A}} + \mathrm{j}\cdot\mu_0\omega\gamma\underline{\boldsymbol{A}} = \mu_0\underline{\boldsymbol{J}}_{\mathrm{ext}} \qquad (1)$$

where ω is the angular frequency, γ the electrical conductivity, $\mu_0 = 4\pi\times10^{-7}\,\mathrm{Hm}^{-1}$ the permeability of vacuum and $\underline{\boldsymbol{J}}_{\mathrm{ext}}$ the phasor of the external current density (calculated from the field currents in the coils). The boundary conditions both along the axis z and artificial boundary sufficiently distant from the system under investigation are of the Dirichlet type ($\underline{\boldsymbol{A}} = \boldsymbol{0}$) .

Eddy current densities and specific average Joule losses in melt are given by formulas

$$\underline{\boldsymbol{J}}_{\mathrm{eddy}} = \mathrm{j}\cdot\omega\gamma\underline{\boldsymbol{A}}\,, \qquad (2)$$

$$w_{\mathrm{J}} = \frac{\left|\boldsymbol{J}_{\mathrm{eddy}}\right|^2}{\gamma}. \quad (3)$$

The specific Lorentz forces acting in melt may be expressed as

$$\boldsymbol{f} = \boldsymbol{J}_{\mathrm{eddy}} \times \boldsymbol{B} \quad (4)$$

where $\boldsymbol{B} = \mathrm{curl}\,\boldsymbol{A}$.

The nonstationary temperature field is only calculated inside the heated body and ceramic pipe. Its distribution in the moving liquid medium obeys the heat transfer equation in the form [5]

$$\begin{aligned} \mathrm{div}\left(\lambda \cdot \mathrm{grad}\, T\right) &= \rho c \cdot \frac{\mathrm{d}T}{\mathrm{d}t} - w_{\mathrm{J}} = \\ &= \rho c \cdot \left(\frac{\partial T}{\partial t} + \boldsymbol{v} \cdot \mathrm{grad}\, T\right) - w_{\mathrm{J}} \end{aligned} \quad (5)$$

where λ denotes the thermal conductivity, ρ the specific mass of the heated material, c its specific heat, $\boldsymbol{v}$ the velocity and w_{J} is given by (3). The boundary conditions along the outer wall of the ceramic pipe of poor thermal conductivity are supposed to be of the Dirichlet type, while at both ends of the investigated part of the pipe are of the Neumann type.

The field of flow is calculated only in the domain of melt. This field is characterized by velocity $\boldsymbol{v}$ and pressure p. Distribution of these quantities is described by the Navier-Stokes equation [1]

$$\rho \cdot \left[\frac{\partial \boldsymbol{v}}{\partial t} + \left(\boldsymbol{v} \cdot \mathrm{grad}\right)\boldsymbol{v}\right] = -\mathrm{grad}\, p + \rho \boldsymbol{g} + \eta \cdot \Delta \boldsymbol{v} + \boldsymbol{f} \quad (6)$$

supplemented with the equation of continuity for incompressible liquids

$$\mathrm{div}\, \boldsymbol{v} = 0. \quad (7)$$

Here the symbol p stands for the pressure, η is the dynamic viscosity and $\boldsymbol{f}$ the vector of the internal volume Lorentz force given by (4). The boundary conditions follow from the character of the task. The normal and tangential components of velocity are supposed to vanish along the walls of the pipe. The velocity of melt and its pressure at the inlet of the pipe are supposed to be known.

3. Numerical solution of the task

The numerical solution of the task is realized by a higher-order finite element method. Computation of electromagnetic field is carried out independently of the temperature and flow fields. The velocity components in the radial and axial directions (v_r, v_z), pressure p, and temperature T are approximated on four different meshes that are adapted to respect the individual behavior of these physical fields. This approach leads to a significant reduction of both the size and conditioning of the discrete problem compared to the discretization on a single mesh.

The meshes for all mentioned fields are *hp*-adaptive. This means such refinement of the mesh where every element can be either *h*-refined (split in space into smaller elements of the same polynomial degree), *p*-refined (its polynomial degree increases), or split in space with arbitrary polynomial degrees in the element sons. This procedure is profoundly different from standard *h* or *p* adaptivity. Its implementation is highly nontrivial, but it may be simplified by making the element refinements local with the help of arbitrary-level hanging nodes [6].

The multi-mesh assembling procedure differs from the standard single-mesh case (see, for example, [7]) in several aspects. For example, the reference maps are slightly more complicated, and one has to take care about the transfer of data among the meshes. We solve this problem by considering a united mesh which is a geometrical union of all meshes. This mesh is never created physically, but the element-by-element assembling procedure parses its virtual elements in a similar way as if all fields were discretized on the united mesh. The transfer of functions from all meshes to the united mesh is fully explicit and very fast. Compared to the discretization of all fields on the united mesh, the multi-mesh assembling procedure is slightly slower due to transfer of these data. However, this all leads to a discrete problem which is smaller and better conditioned.

4. Conclusion

The full paper will contain more detailed mathematical model and also more information about its numerical solution. Several arrangements will be investigated differing by the distribution of the field coils along the pipe and their currents. The results will be compared.

Attention will also be paid to the mathematical aspects of the problem such as accuracy and stability of computations.

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