

# Modelling of the Impact of Phase Asymmetries on the Torque Waveform of a Permanent Magnet Synchronous Motor

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**Abstract.** This paper presents the modelling of asymmetries of phase parameters (inductances) of permanent magnet (PM) synchronous motor and the analysis of the impact of these asymmetries on the instantaneous torque. The torque ripple created by phase asymmetries is simulated for three operating conditions – current-fed, voltage-fed and vector control operation, using specifically developed mathematical model of 3-phase asymmetrical machine.

## Key words

Permanent magnet synchronous motor, phase parameter asymmetries, 3-phase asymmetrical model, torque ripple.

## 1. Introduction

It is usually assumed that manufactured 3-phase rotary machines have symmetrical windings. However, because of tolerances in the manufacturing process the asymmetries between phases may occur. The causes of phase asymmetries can be of mechanical or electromagnetic nature. For instance, the rotor eccentricity causes variation of airgap [1] which is reflected in the asymmetries of phase inductances. The asymmetries between leakage inductances can originate from asymmetries of the winding heads (parts which are out of the magnetic core), and due to possible differences in the distribution of the coil conductors of different slots [2]-[4]. Furthermore, the linear machines have inherent asymmetries between phases, which are associated with increased reluctance of flux paths at the core ends.

The dynamic analysis of 3-phase machines is usually done using the d-q model which assumes that the machine parameters (inductances) of different phases are symmetrical. In [5], the d-q model is developed to study the performance of vector controlled unbalanced 3-phase system. Two complex-conjugate block-structures were introduced to represent the phase asymmetries. An alternative method [6] of modelling the PM synchronous machine with phase asymmetries, introduced a magnetic

circuit model for specifying the flux paths associated with asymmetrical phases.

However, the mathematical models of PM synchronous motors are inherently nonlinear, and introducing further supplements makes the models more cumbersome for applications in control algorithms.

This paper describes the model of PM synchronous machine with asymmetrical phases based on the original 3-phase circuit diagram and presents the waveforms of instantaneous torque which were predicted for the purpose of comparing the torque ripple for three operating conditions: current-fed, voltage-fed and vector control operation.

## 2. Mathematical model of PM synchronous machine with asymmetrical phases

The following assumptions have been made in the model:

- 1) the machine is not in a fault condition,
- 2) the phase windings are star connected (3 output terminals),
- 3) magnetic saturation is neglected, i.e. inductances are independent of current,
- 4) core losses are negligible,
- 5) the back e.m.f. is sinusoidal and
- 6) the cogging torque due to the slotted structure is negligible.

Fig. 1 represents the circuit diagram of conventional a.c. PM synchronous motor, which is used as reference. Commonly, the Park's transformation of 3-phase into d-q system (d-q model) is applied for dynamic analyses of symmetrical PM synchronous machines, because in a reference introduced the transformation of machine parameters is far more complex.

In the study presented below, the model of asymmetrical machine is based on the original (untransformed) 3-phase circuit diagram shown in Fig. 1, where the usual conventions are applied.

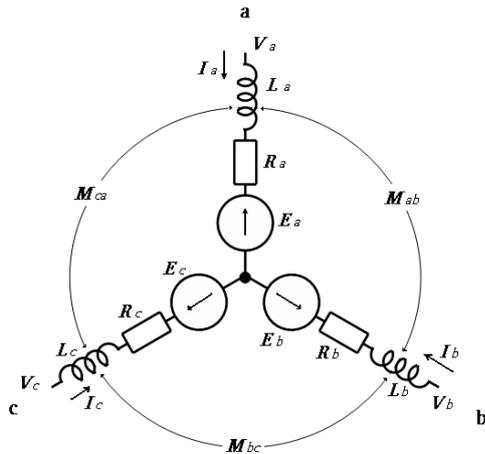


Fig.1. Circuit diagram of PM synchronous motor

Under sinusoidal current-fed operation the instantaneous torque is obtained by solving the torque and position equations simultaneously.

Under sinusoidal voltage-fed operation, the circuit differential equations together with the torque and position equations are integrated numerically using the method described in [7] to obtain the instantaneous current and torque waveforms.

An example of the impact of phase asymmetries on the instantaneous torque is depicted in Figs. 2 to 4 which relate to the current-fed, voltage-fed and vector control operation of the 6-pole PM synchronous motor at the speed of 36 rev/min (supply frequency 1.8 Hz). In this example the phase asymmetry is caused by the difference (20% reduction) of the flux-linkage between one and the rotor compared to the other two phases. Such an asymmetry results in the appearance of the torque ripple of double frequency which has magnitude 1.2% of the average torque for current-fed operation, and 1.5% for the voltage-fed operation. In these two modes the system operates as in an open loop state with respect to the rotor position (self-adjusting load angle) and the torque ripple is similar for both modes. For vector control operation the torque ripple is larger (9%) due to the load angle being locked to the predefined value.

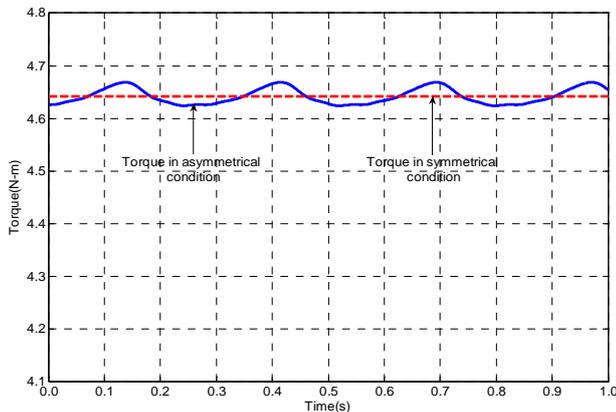


Fig.2. Torque waveform in voltage-fed operation

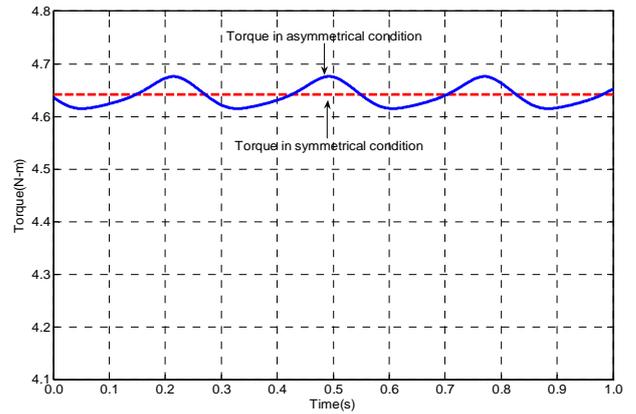


Fig.3. Torque waveform in voltage-fed operation

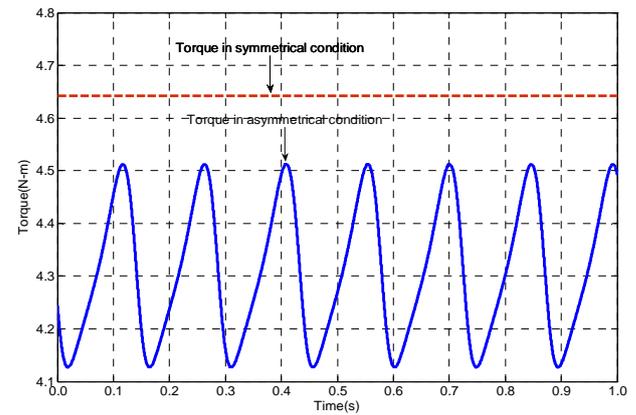


Fig.4. Torque waveform in vector control operation

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