Modelling of the Impact of Phase Asymmetries on the Torque Waveform of a Permanent Magnet Synchronous Motor

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Abstract. This paper presents the modelling of asymmetries of phase parameters (inductances) of permanent magnet (PM) synchronous motor and the analysis of the impact of these asymmetries on the instantaneous torque. The torque ripple created by phase asymmetries is simulated for three operating conditions – current-fed, voltage-fed and vector control operation, using specifically developed mathematical model of 3-phase asymmetrical machine.

Key words

Permanent magnet synchronous motor, phase parameter asymmetries, 3-phase asymmetrical model, torque ripple.

1. Introduction

It is usually assumed that manufactured 3-phase rotary machines have symmetrical windings. However, because of tolerances in the manufacturing process the asymmetries between phases may occur. The causes of phase asymmetries can be of mechanical or electromagnetic nature. For instance, the rotor eccentricity causes variation of airgap [1] which is reflected in the asymmetries of phase inductances. The asymmetries between leakage inductances can originate from asymmetries of the winding heads (parts which are out of the magnetic core), and due to possible differences in the distribution of the coil conductors of different slots [2]-[4]. Furthermore, the linear machines have inherent asymmetries between phases, which are associated with increased reluctance of flux paths at the core ends.

The dynamic analysis of 3-phase machines is usually done using the d-q model which assumes that the machine parameters (inductances) of different phases are symmetrical. In [5], the d-q model is developed to study the performance of vector controlled unbalanced 3-phase system. Two complex-conjugate block-structures were introduced to represent the phase asymmetries. An alternative method [6] of modelling the PM synchronous machine with phase asymmetries, introduced a magnetic circuit model for specifying the flux paths associated with asymmetrical phases.

However, the mathematical models of PM synchronous motors are inherently nonlinear, and introducing further supplements makes the models more cumbersome for applications in control algorithms.

This paper describes the model of PM synchronous machine with asymmetrical phases based on the original 3-phase circuit diagram and presents the waveforms of instantaneous torque which were predicted for the purpose of comparing the torque ripple for three operating conditions: current-fed, voltage-fed and vector control operation.

2. Mathematical model of PM synchronous machine with asymmetrical phases

The following assumptions have been made in the model:

1) the machine is not in a fault condition,
2) the phase windings are star connected (3 output terminals),
3) magnetic saturation is neglected, i.e. inductances are independent of current,
4) core losses are negligible,
5) the back e.m.f. is sinusoidal and
6) the cogging torque due to the slotted structure is negligible.

Fig.1 represents the equivalent circuit diagram of 3-phase PM synchronous motor, which is used as a reference. Usually, the Park's transformation of 3-phase into d-q model is applied for dynamic analyses of symmetrical PM synchronous machines. However, when asymmetries are introduced the transformation of machine parameters is far more complex.

In the study presented below, the model of asymmetrical machine is based on the original (untransformed) 3-phase circuit diagram shown in Fig. 1, where the usual conventions are applied i.e. the stator reference axis is chosen to coincide with the phase *a*, and the phases *b* and *c* are 120° and 240° respectively behind the phase *a*.

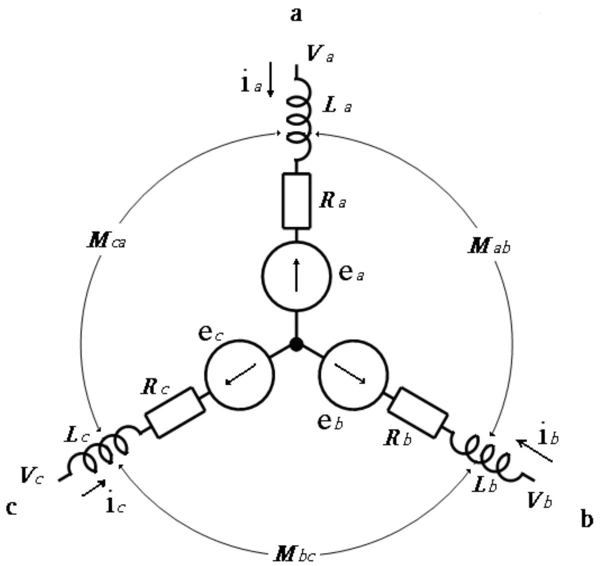

Fig. 1. Circuit diagram of PM synchronous motor

According to Fig. 1, the circuit equations are given as follows:-

$$v_{ab} = L_a \frac{di_a}{dt} + M_{ab}\frac{di_b}{dt} + M_{ac}\frac{di_c}{dt} + e_a + R_a i_a - M_{ba}\frac{di_a}{dt} - L_b\frac{di_b}{dt} - M_{bc}\frac{di_c}{dt} - e_b - R_b i_b \tag{1}$$

$$v_{bc} = M_{ba}\frac{di_a}{dt} + L_b\frac{di_b}{dt} + M_{bc}\frac{di_c}{dt} + e_b + R_b i_b - M_{ca}\frac{di_a}{dt} - M_{cb}\frac{di_b}{dt} - L_c\frac{di_c}{dt} - e_c - R_c i_c \tag{2}$$

$$v_{ca} = M_{ca}\frac{di_a}{dt} + M_{cb}\frac{di_b}{dt} + L_c\frac{di_c}{dt} + e_c + R_c i_c - L_a\frac{di_a}{dt} - M_{ab}\frac{di_b}{dt} - M_{ac}\frac{di_c}{dt} - e_a - R_b i_b \tag{3}$$

$$i_a + i_b + i_c = 0 \tag{4}$$

$$e_a = \frac{d\psi_{af}}{dt}; \qquad \psi_{af} = \Psi_{af}\cos(\frac{n_p}{2}\alpha) \tag{5}$$

$$e_b = \frac{d\psi_{bf}}{dt}; \qquad \psi_{bf} = \Psi_{bf}\cos(\frac{n_p}{2}\alpha - 120°) \tag{6}$$

$$e_c = \frac{d\psi_{cf}}{dt}; \qquad \psi_{cf} = \Psi_{cf}\cos(\frac{n_p}{2}\alpha + 120°) \tag{7}$$

where Ψ_{af}, Ψ_{bf} and Ψ_{cf} denote maximum values of mutual flux linkages between each stator phase and the rotor; R_a, R_b and R_c are phase resistances; L_a, L_b and L_c are phase self-inductances; M_{ab}, M_{bc} and M_{ca} are mutual inductances between phases; α denotes the instantaneous position of the rotor axis related to axis of phase *a,* and n_p is the number of poles.

The electromagnetic torque is given as:-

$$T_e = i_a \frac{d\psi_{af}}{d\alpha} + i_b \frac{d\psi_{bf}}{d\alpha} + i_c \frac{d\psi_{cf}}{d\alpha} \tag{8}$$

The equations of dynamic motion are:-

$$T_e - T_{load} = J\frac{d\omega}{dt} + D\omega \tag{9}$$

$$\omega = \frac{d\alpha}{dt} \tag{10}$$

where J and D denote the moment of inertia and the coefficient of viscous friction, T_{load} represents the load torque and ω denotes the rotor speed.

Assuming the phase asymmetry is caused by a reduction of the flux linkage between phase *a* and the rotor, relative to the other two phases, i.e.

$$\Psi_{af} = (1-\xi)\Psi_f, \quad \Psi_{bf} = \Psi_f, \quad \Psi_{cf} = \Psi_f$$

where ξ denotes the factor of flux linkage reduction.

A. Current-fed operation

For current-fed operation without position feedback, the phase currents are independent and mutually balanced.

$$i_a = I_m \sin(\omega_e t) \tag{11}$$

$$i_b = I_m \sin(\omega_e t - 120°) \tag{12}$$

$$i_c = I_m \sin(\omega_e t + 120°) \tag{13}$$

where I_m denotes the amplitude of phase current and ω_e is the electrical angular frequency of current source ($\omega_e = 2\pi f$).

Substituting equations (11) to (13) into equation (8), results in the following expression for the torque:-

$$T_e = I_m \sin(\omega_e t)\frac{d\psi_{af}}{d\alpha} + I_m \sin(\omega_e t - 120°)\frac{d\psi_{bf}}{d\alpha} + I_m \sin(\omega_e t + 120°)\frac{d\psi_{cf}}{d\alpha} \tag{14}$$

This expression can be transformed into:-

$$T_e = -\frac{n_p}{2} I_m \Psi_f \frac{3}{2}\cos(\omega_e t - \frac{n_p}{2}\alpha) - \frac{n_p}{2} \times \Psi_f \times \xi \times I_m \sin(\omega_e t)\sin(\frac{n_p}{2}\alpha) \quad (15)$$

The second part of the last expression can be decomposed into two segments, i.e.:

$$T' = -\frac{n_p}{4} \times \Psi_f \times \xi \times I_m \cos(\omega_e t - \frac{n_p}{2}\alpha) \quad (16)$$

$$T'' = -\frac{n_p}{4} \times \Psi_f \times \xi \times I_m \cos(\omega_e t + \frac{n_p}{2}\alpha) \quad (17)$$

The equation (15), together with equations (9) and (10), are integrated simultaneously to obtain instantaneous torque variation in time.

B. *Voltage-fed operation*

Under sinusoidal voltage-fed operation the supply voltages are:

$$v_{ab} = V_m \sin(\omega_e t) \quad (18)$$

$$v_{bc} = V_m \sin(\omega_e t - 120^\circ) \quad (19)$$

$$v_{ca} = V_m \sin(\omega_e t + 120^\circ) \quad (20)$$

where V_m denotes the line voltage amplitude.

Combining the equations (1) to (7) and (18) to (20), results in the set of equations which is written in the matrix form as:-

$$\begin{bmatrix} (L_a - M_{ba}) + \frac{R_b}{R_b+R_c}(M_{ba} - M_{ca} - M_{bc} + L_c) & (M_{ba} - L_b) + \frac{R_b}{R_b+R_c}(L_b - M_{cb} - M_{bc} + L_c) & M_{ac} - M_{bc} \\ M_{ba} - M_{ca} & (L_b - M_{cb}) + \frac{R_c}{R_c+R_a}(M_{cb} - M_{ca} - M_{ab} + L_a) & (M_{bc} - L_c) + \frac{R_c}{R_c+R_a}(L_a - M_{ca} - M_{ac} + L_c) \\ (M_{ca} - L_a) + \frac{R_a}{R_a+R_b}(L_a - M_{ba} - M_{ab} + L_b) & M_{cb} - M_{ab} & (L_c - M_{ac}) + \frac{R_a}{R_a+R_b}(M_{ac} - M_{ab} - M_{bc} + L_b) \end{bmatrix} \times \begin{bmatrix} \frac{di_a}{dt} \\ \frac{di_b}{dt} \\ \frac{di_c}{dt} \end{bmatrix}$$

$$= \begin{bmatrix} v_{ab} + \frac{R_b}{R_b+R_c} v_{bc} - e_a - \left(\frac{R_b}{R_b+R_c} - 1\right) e_b + \frac{R_b}{R_b+R_c} e_c - \left(R_a + \frac{R_b R_c}{R_b+R_c}\right) i_a \\ v_{bc} + \frac{R_c}{R_c+R_a} v_{ca} - e_b - \left(\frac{R_c}{R_c+R_a} - 1\right) e_c + \frac{R_c}{R_c+R_a} e_a - \left(R_b + \frac{R_c R_a}{R_c+R_a}\right) i_b \\ v_{ca} + \frac{R_a}{R_a+R_b} v_{ab} - e_c - \left(\frac{R_a}{R_a+R_b} - 1\right) e_a + \frac{R_a}{R_a+R_b} e_b - \left(R_c + \frac{R_a R_b}{R_a+R_b}\right) i_c \end{bmatrix} \quad (21)$$

The matrix set of electrical equations (21), together with equations (8) to (10), are integrated numerically using the method described in [7] to obtain the instantaneous current and torque variations in time.

C. *The vector control operation*

Fig. 2 shows the block diagram of the conventional vector control scheme [8]. The matrix set of electrical equations (21), together with equations (8) to (10), is embedded in the mathematical model of vector control and which enables prediction of the instantaneous torque variation in time.

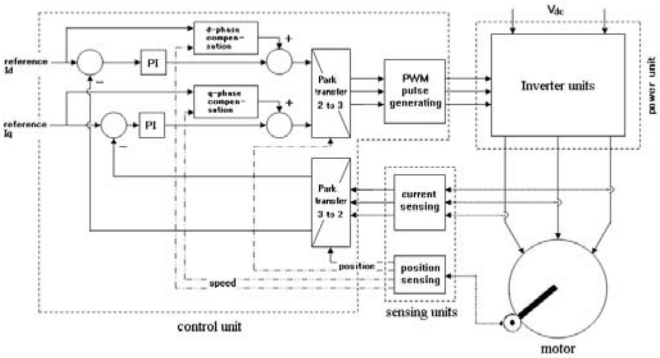

Fig. 2. Vector control block diagram

3. Predictions of Torque Ripple Caused by Phase Asymmetries

The impact of phase asymmetries on the instantaneous torque has been examined on a 6-pole, 180 Hz, 294 V (line), 1.73 kW permanent magnet synchronous motor, which has been considered to have a hypothetical asymmetry caused by a 20% reduction of the flux linkage between phase *a* and the rotor, Ψ_{af}, relative to the other two phases.

Open loop (voltage-fed and current-fed) operation

The torque ripple was predicted for a low speed operation. When the motor was running in an open loop state, such as the voltage-fed or the current-fed operation, the operating frequency was given the value of 1.8 Hz which corresponds to the synchronous speed of 36 rev/min.

For voltage-fed operation at this frequency the line voltage was set to 10 V, so that the flux was kept the rated level. The average torque achieved in this operating state was 4.64 Nm. The torque ripple is depicted in Fig.3.

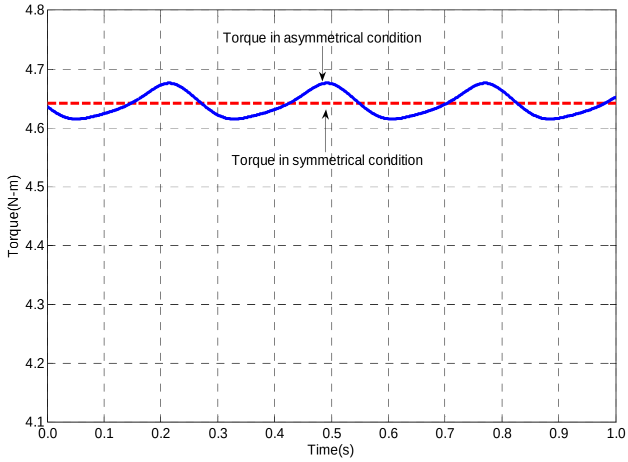

Fig. 3. Torque waveform in voltage-fed operation

Under the current-fed condition the same average torque is achieved at the current of 9.5 A. The torque ripple for such an operation is depicted in Fig.4.

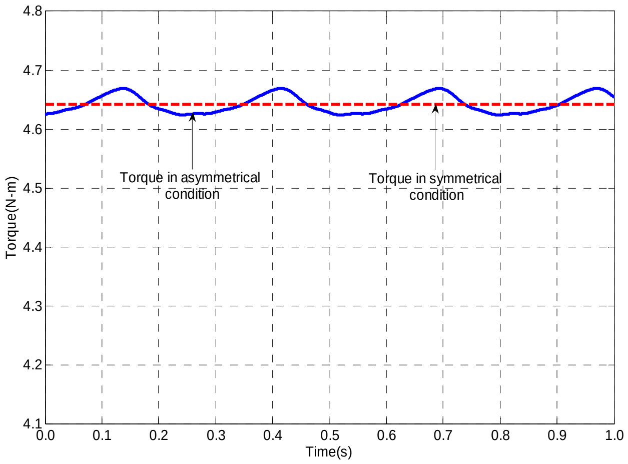

Fig. 4. Torque waveform in current-fed operation

In both the voltage-fed and the current-fed operation the hypothetical 20% asymmetry in the flux linkage of one phase relative to the other two, results in the appearance of a non-sinusoidal torque ripple whose fundamental harmonic has double frequency relative to the operational frequency. The peak-to-peak magnitude of the ripple is 1.5% of the average torque for the voltage-fed operation and 1.2% for the current-fed operation.

Vector control operation

In vector control operation the currents i_d and i_q were given the values determined from the amplitude value of the current and the load angle occurring under the voltage-fed operation without phase asymmetries. The torque ripple is depicted in Fig.5. The peak-to-peak magnitude of the ripple is now 9% of the average torque and the average torque has a reduced value by 0.35 Nm (i.e. 7.5%) compared to the symmetrical condition. because the load angle is locked and remains unchanged compared to the one specified under the symmetrical condition.

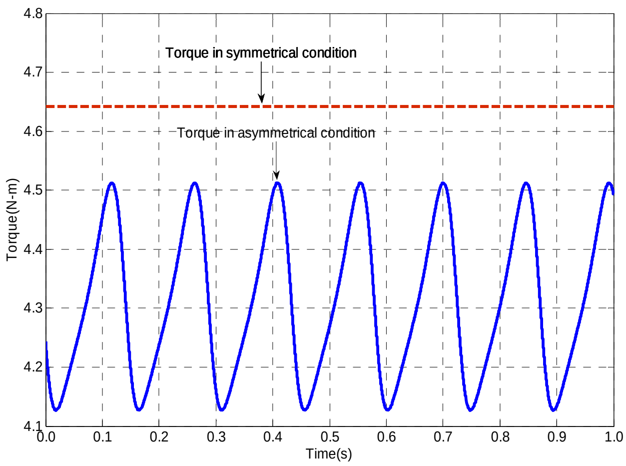

Fig. 5. Torque waveform in vector control operation

4. Conclusions

The mathematical model of PM motor with asymmetrical phase parameters has been developed with the aim to analyse the impact of the asymmetries on the variation of instantaneous torque. The torque ripple was predicted for three operating modes: current-fed, voltage-fed and vector control operation. In the first two modes the system operates as in an open loop state with respect to the rotor position (self-adjusting load angle) and the torque ripple is similar for both modes. In the vector control mode, the system operates in a position-closed loop state and the torque ripple is larger compared with the former two modes due to the load angle being locked to the predefined value.

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