Dynamic Phasors Modeling of the Wound Rotor Induction Generator for Electromagnetic and Electromechanical Analysis

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Abstract. Decentralized generation and micro grids are more common in new electric power systems, where there are much kind of equipments like wind energy conversion systems, PV systems, storage systems, electronic converters that transfer the energy produce by these renewable energies and FACTS that are implemented in the whole system to guaranteed stability and quality of electric parameters. Some of these subsystems operate in continuous mode and others in discrete mode. For reasons mentioned above, it is very important to develop models of these technologies that let to analyze theirs dynamics, both in short as in long periods of time. In this work we use dynamic phasors methodology to model the wound rotor induction generator in DQ0 reference frame and we analyze electromagnetic and electromechanical dynamics where simulation results demonstrate good approximation of real states with mechanical load, stator voltage amplitudes and frequency variations. Dynamic phasors have the characteristic to include harmonics in the models, so they can simulate complex nonlinear systems and big power systems in an accurate and efficient way and constitute a useful simulation tool, that fill the gap between Electromagnetic Transient Programs and Transient Stability Programs.

Key words

Dynamic phasors, wound rotor induction generator, electromagnetic and electromechanical analysis, renewable energy.

1. Introduction

Renewable energies have been object of major attention in the whole World. Some reasons are advances in technologies that have made them more achievable in different ways; we only mention four of them: cost, efficiency, reliability and quality.

First point is obtained due to mass production that permits low cost/kW generated. Secondly, power electronics join with other research areas like materials science, have increased the efficiency in wind energy conversion system (WECS), PV systems and storage devices for example, permitting them to work in optimal values (MPPT). These two points are right now some of the most important requirements to implement these kinds of technologies in power electric system (e.g. distributed generation forming or not microgrids).

In the last years WECS are being installed widely, and the wound rotor induction generator (WRIG) is one of the most popular electric machines because of its flexibility of operation. These systems present electromagnetic and electromechanical dynamics which are very complex to analyse due to the different duration. To cover this necessity, different models have been developed to simulate them and be able to diagnostic electric system "health" and prevent blackouts or big variations in the parameters as voltages sags, dips, powers distortions, etc. These models are made in electromagnetic transient programs (EMTP) and transient stability programs (TSP) that is a quasi stationary analysis, but there is an area that is not covered for any method: simulate both electromagnetic and electromechanical dynamics in high scale systems with accuracy and efficiency, to be able to take decisions and guaranteed stability and quality of power system [1].

There are many works about rotating machines models, as in [1] a doubly fed induction generator is modelling with dynamic phasors, considering dc component and second harmonic for all variables. Also the converter and the transformer models are obtained. All equations are expressed in general form, and are developed in DQ rotor reference frame. A comparative assessment of detailed, fundamental frequency, dynamic phasors, and reduced order dynamic phasors models is done.

In [2] a three phase self-excited induction generator is modelled in state space, for the purpose to analyse transient response when capacitance and electrical load variations are applied.

A model for a single-phase induction machine is developed with dynamic phasors in [3], where keep first harmonic for currents and dc component and second harmonic for speed. In this work a small signal analysis is done, where eigenvalues are obtained for the 9^{th} order and 7^{th} reduced models.

In [4] a dynamic phasors model is obtained for a three phase induction motor and for a permanent magnet synchronous machine. In both cases equations are not presented in state space, but as stator and rotor voltages. In first case is mentioned that transient dynamic with unbalanced voltages is explored experimentally and numerically. However, the work does not present any simulation for the induction motor dynamic, but these are presented for the permanent magnet synchronous machine, where can be seen that variables dynamic of the proposed model have grate accuracy.

Other applications with dynamic phasors technique have been done, like modelling of arcing faults on overhead lines in [5], a hybrid-model transient stability simulation of HVDC transmission system is done in [6], and analysis of balanced and unbalanced faults in power systems in [7]. In these three works the first conclusion is how dynamic phasors modelling is a very good approximation of detailed time-domain models like EMTP programs, and with shorter simulation time.

Another area of widespread application is in power electronics [8]-[14], where power quality analysis is studied for STATCOM, series resonant converters, dynamic voltage restorer, unified power flow controller, thyristor-controlled series capacitor, static VAR compensator, and general pulse modulated systems. In these works the common interest is to analyze the transient dynamic under unbalanced conditions with symmetrical and asymmetrical faults. A key consideration is which *k-th* Fourier coefficients to take into account to obtain an accuracy analysis, for example in some application are selected k=1 when the fundamental frequency is dominant, as in currents and voltages for induction machines, k=0, 2 for mechanical speed and dc-dc converters, where the DC component describes very well the dynamics of the original signal and the second harmonic provides further information on the oscillatory dynamics. Other authors choose k=1, 3 and 5 to analyze harmonic content presented in the system under variations mentioned above, depending on the nature of the system.

This work present a DQ dynamic phasors model of the wound rotor induction generator, with four equations to describe stator and rotor currents dynamics, that include the first Fourier coefficient, this is at their fundamental frequency, and two equations to describe the rotor electrical speed, one includes dc component and the other of the second harmonic. This model is useful to analysis dynamics when there are load, voltages amplitude and frequency variations. Active and reactive power dynamic phasors equations are presented. Simulation results in Matlab/Simulink show the accuracy of the proposed model.

2. Outlines of the dynamic phasors approach

In recent years dynamic phasors are being used for modeling different elements of power electric system, as generators, power electronics converters, transmission lines, transformers, loads, and Flexible AC Transmission System (FACTS). The main idea of dynamic phasors approach is to approximate a possibly complex time domain waveform $x(\tau)$ in the interval $\tau \in (\tau - T, t]$ with a Fourier series representation of the form [9]:

$$x(\tau) \approx \sum_{-\infty}^{\infty} X_k(t) \cdot e^{jk\omega\tau} \qquad (1)$$

$$X_k(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau) \cdot e^{\cdot jk\omega\tau} d\tau = \langle x \rangle_k (t) \qquad (2)$$

Where $\omega = 2\pi/T$ and $X_k(t)$ is the k_{th} time varying Fourier coefficient in complex form, also called dynamic phasors, *k* is the set of selected Fourier coefficients which provide a good approximation of the original waveform (e.g. *k*=0, 1, 2) and *j* is the imaginary operator. Some important properties of dynamic phasors are: the relation between the derivatives of $x(\tau)$ and the derivatives of $X_k(t)$, which is given in (3). This is obtained differentiating (1)

$$\left\langle \frac{dx}{dt} \right\rangle_k = \frac{dX_k}{dt} - jk\omega X_k \qquad (3)$$

The product of two time-domain variables equals a discrete time convolution of the two dynamic phasors sets of variables, which is given in (4).

$$\langle xy \rangle_k = \sum_{l=-\infty}^{\infty} (X_{k-l} Y_l) \qquad (4)$$

In this paper we focus in developing and analyzing the behavior of the WRIG with the two methodologies mentioned above, the instantaneous time domain model, and the dynamic phasors. Both are done in DQ0 reference frame.

3. WRIG models

In this section we first model the WRIG in DQ0 reference frame, and from this we develop the dynamic phasors model.

A. WRIG Transient model.

We started with the electrical diagrams shown in figure 1, and with $\boldsymbol{\omega} = \mathbf{0}$ we derived the equations of magnetic

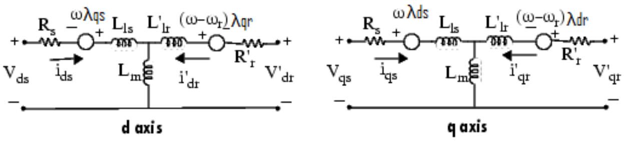


Fig.1. Electrical diagrams of the induction machine

fluxes and voltages in the stationary reference frame show in (5) and (6), that describe the instantaneous dynamic of the machine [**15**] .

$$\begin{aligned} \mathbf{V_{ds}} &= \mathbf{R_s i_{ds}} + \frac{\mathbf{d}}{\mathbf{dt}}\boldsymbol{\lambda}_{\mathbf{ds}} \qquad \boldsymbol{\lambda}_{\mathbf{ds}} = \mathbf{L_s i_{ds}} + \mathbf{L_m i_{dr}} \\ \mathbf{V_{qs}} &= \mathbf{R_s i_{qs}} + \frac{\mathbf{d}}{\mathbf{dt}}\boldsymbol{\lambda}_{\mathbf{qs}} \qquad \boldsymbol{\lambda}_{\mathbf{qs}} = \mathbf{L_s i_{qs}} + \mathbf{L_m i_{qr}} \\ \mathbf{V_{dr}} &= \mathbf{R_r i_{dr}} + \frac{\mathbf{d}}{\mathbf{dt}}\boldsymbol{\lambda}_{\mathbf{dr}} + \boldsymbol{\omega}_{\mathbf{r}}\boldsymbol{\lambda}_{\mathbf{qr}} \qquad \boldsymbol{\lambda}_{\mathbf{dr}} = \mathbf{L_r i_{dr}} + \mathbf{L_m i_{ds}} \\ \mathbf{V_{qr}} &= \mathbf{R_r i_{qr}} + \frac{\mathbf{d}}{\mathbf{dt}}\boldsymbol{\lambda}_{\mathbf{qr}} - \boldsymbol{\omega}_{\mathbf{r}}\boldsymbol{\lambda}_{\mathbf{dr}} \qquad \boldsymbol{\lambda}_{\mathbf{qr}} = \mathbf{L_r i_{qr}} + \mathbf{L_m i_{qs}} \end{aligned} \tag{5}$$

And electromechanical equation

$$\frac{\mathbf{d}}{\mathbf{dt}}\boldsymbol{\omega}_{\mathbf{r}} = \frac{\mathbf{P}}{\mathbf{J}}\left(\mathbf{T_e} - \frac{\mathbf{B}}{\mathbf{P}}\boldsymbol{\omega}_{\mathbf{r}} - \mathbf{T_L}\right) \tag{6}$$

Where $\mathbf{T_e} = \frac{3}{2}\mathbf{P}(\boldsymbol{\lambda}_{\mathbf{ds}}\mathbf{i_{qs}} - \boldsymbol{\lambda}_{\mathbf{qs}}\mathbf{i_{ds}})$ is the electromagnetic torque. For simplification, only main definitions are mentioned, where subscripts $\boldsymbol{d}$ and q refer to direct and quadrature-axis, $\boldsymbol{s}$ and $\boldsymbol{r}$ refer to stator and rotor parameters respectively. $\boldsymbol{R}$ are resistances, L are inductances, V are voltages, i are currents, $\boldsymbol{\lambda}$ are magnetic fluxes, $\boldsymbol{\omega_r}$ is the electrical speed, P is the number of pole pairs, B is the viscosous friction coefficient, $\boldsymbol{T_L}$ is the mechanical load and $\boldsymbol{L_m}$ is the mutual inductance. It is important to take into account that for implementation purpose is more feasible to meter currents than magnetic fluxes. This is the main reason to develop our model with currents as states variables. Thus, deriving magnetic fluxes equations from 5 and clearing for currents and adding equation 6, we obtain the fifth order nonlinear system equations model in the form of state space that describes the transient dynamic of the WRIG, as is shown in (7).

This model is simulated to validate its performance, and then, with the theory background of section 2, a model with dynamic phasors is developed, as is presented in next subsection.

$$\begin{aligned} \frac{di_{ds}}{dt} &= a_{11}i_{ds} - a_{12}i_{qs}\omega_r - a_{13}i_{dr} - a_{14}i_{qr}\omega_r - b_1v_{ds} + b_2v_{dr} \\ \frac{di_{qs}}{dt} &= a_{12}i_{ds}\omega_r + a_{11}i_{qs} + a_{14}i_{dr}\omega_r - a_{13}i_{dr} - b_1v_{qs} + b_2v_{qr} \\ \frac{di_{dr}}{dt} &= -a_{21}i_{ds} + a_{32}i_{qs}\omega_r + a_{33}i_{dr} + a_{34}i_{qr}\omega_r + b_2v_{ds} - b_3v_{dr} \\ \frac{di_{qr}}{dt} &= -a_{32}i_{ds}\omega_r - a_{31}i_{qs} - a_{34}i_{dr}\omega_r + a_{33}i_{qr} + b_2v_{qs} - b_3v_{qr} \\ \frac{d\omega_r}{dt} &= \frac{P}{J}\left[\frac{3}{2}PL_m\left(i_{qs}i_{dr} - i_{ds}i_{qr}\right) - \frac{B}{P}\omega_r - T_L\right] \end{aligned} \tag{7}$$

Where $\boldsymbol{a_{ij}}, \boldsymbol{b_i}$ are constant parameters defined as:
$a_{11} = KR_sL_r$; $a_{12} = KL_m^2$; $a_{13} = KR_rL_m$; $a_{14} = KL_mL_r$; $a_{31} = KR_sL_m$; $a_{32} = KL_mL_s$; $a_{33} = KR_rL_s$; $a_{34} = KL_sL_r$; $b_1 = KL_r$; $b_2 = KL_m$; $b_3 = KL_s$ with K define as $K = \frac{1}{L_m^2 - L_sL_r}$.

B. Proposed WRIG dynamic phasors model.

To obtain an accurate response of a dynamic phasors model is important to be careful in the selection of Fourier coefficients, because they represent the harmonics of the real signals, and depend of the analysis purpose it should has more or less coefficients. In our case, we choose $k = \pm 1$ for currents, this is at fundamental frequency, and $k = 0,2$ for electrical speed, having a dc and second harmonic components [4].

The obtained model is shown in (8), where $\bar{i}$ represents the conjugate of a complex number. This is a 6th order nonlinear equation model, where states are time-variant complex Fourier coefficients. This model can be represented by 11 nonlinear real equations.

$$\begin{aligned} \frac{di^1_{ds}}{dt} &= (a_{11} - j\omega_s)i^1_{ds} - a_{12}(i^1_{qs}\omega^0_r + \bar{i}^{-1}_{qs}\omega^2_r) - a_{13}i^1_{dr} \\ &\quad - a_{14}(i^1_{qr}\omega^0_r + \bar{i}^{-1}_{qr}\omega^2_r) - b_1v_{ds} + b_2v_{dr} \\ \frac{di^1_{qs}}{dt} &= a_{12}(i^1_{ds}\omega^0_r + \bar{i}^{-1}_{ds}\omega^2_r) + (a_{11} - j\omega_s)i^1_{qs} \\ &\quad + a_{14}(i^1_{dr}\omega^0_r + \bar{i}^{-1}_{dr}\omega^2_r) - a_{13}i^1_{dr} - b_1v_{qs} + b_2v_{qr} \\ \frac{di^1_{dr}}{dt} &= -a_{21}i^1_{ds} + a_{32}(i^1_{qs}\omega^0_r + \bar{i}^{-1}_{qs}\omega^2_r) + (a_{33} - j\omega_s)i^1_{dr} \\ &\quad + a_{34}(i^1_{qr}\omega^0_r + \bar{i}^{-1}_{qr}\omega^2_r) + b_2v_{ds} - b_3v_{dr} \\ \frac{di^1_{qr}}{dt} &= -a_{32}(i^1_{ds}\omega^0_r + \bar{i}^{-1}_{ds}\omega^2_r) - a_{31}i^1_{qs} \\ &\quad - a_{34}(i^1_{dr}\omega^0_r + \bar{i}^{-1}_{dr}\omega^0_r) + (a_{33} - j\omega_s)i^1_{qr} + b_2v_{qs} - b_3v_{qr} \\ \frac{d\omega^0_r}{dt} &= \frac{P}{J}\left[\frac{3}{2}PM\left(i^1_{qs}\bar{i}^{-1}_{dr} - i^1_{ds}\bar{i}^{-1}_{qr}\right) - B\omega^0_r - T_L\right] \\ \frac{d\omega^2_r}{dt} &= \frac{P}{J}\left[\frac{3}{2}PM\left(i_{qs}i_{dr} - i_{ds}i_{qr}\right) - B\omega^2_r\right] - j\omega_s\omega^2_r \end{aligned} \tag{8}$$

4. Results

In order to validate both models, these were simulated in Matlab/Simulink with a fixed step of 0.0001 seconds and Runge-Kutta solution method. Results were compared with asynchronous machine developed in SimPower Systems library, as is presented in figure 2. Parameters values of the machine are: 7.5 kW, 400 V, 50 Hz, 1440 RPM, $\mathbf{R_s = 0.7384\Omega}$, $\mathbf{R_r = 0.7402\Omega}$, $\mathbf{L_s = L_r = 0.1271H}$, $\mathbf{J = 0.0343}$, $\mathbf{P = 2}$, $\mathbf{B = 0.000503}$.

C. Fourier coefficients dynamic response analysis.

Dynamic response of electrical and mechanical variables is presented in figures 3-7. In figure 3, original and approximated dynamics from equation 7 and 8 are shown, respectively. It can be seen that module current, which is calculated from equation 9, follows the original signal when a step change is applied in mechanical load from -50 to -100 Nm at 0.4 seconds.

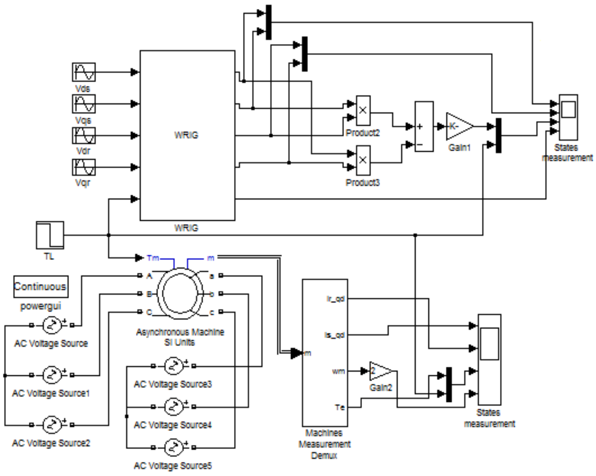


Fig.2. Wound rotor induction generator block diagram developed in Matlab/Simulink.

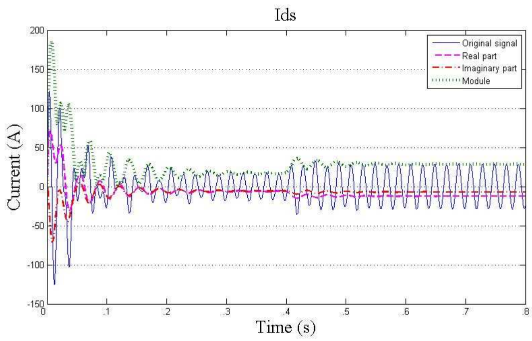


Fig. 3. Stator direct-axis current dynamic.

Also is shown how both components of Fourier coefficients, real and imaginary parts, are negative in steady state. However in figure 4 q-axis rotor current components have opposite sign, but its module current has similar dynamic as in previous case.

$$|i_{ds}| = 2\sqrt{(i_{ds}^{re})^2 + (i_{ds}^{im})^2} \tag{9}$$

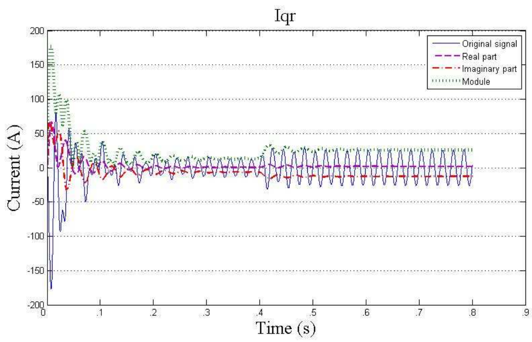


Fig. 4. Rotor quadrature-axis current dynamic.

Torque dynamic is presented in figure 5, where can be seen how the original signal is very close estimated by electromagnetic torque dc component dynamic phasor.

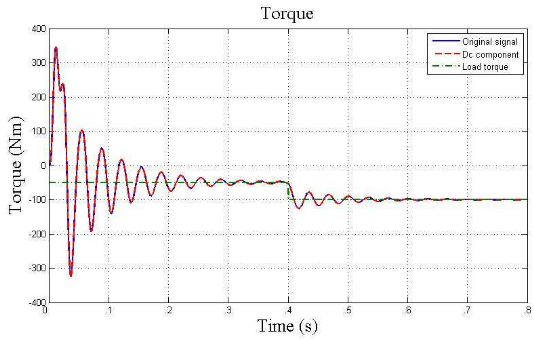


Fig.5. Torque dynamic.

When the machine is operated in generation mode, this is with negative torque applied to the shaft, by a wind mill for example, mechanical speed is in super synchronous operation, (> synchronous speed of 1500 rpm for our machine), as is observed in figure 6, where the original signal is followed by dc component dynamic phasors.

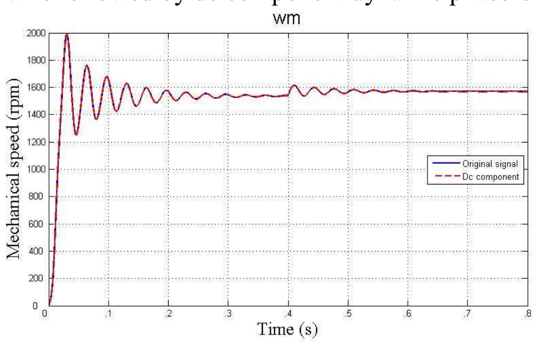


Fig.6. Mechanical speed dynamic.

In figure 7 is presented how the original signal is approximated by dynamic phasors coefficients, when a step variation in mechanical load $\boldsymbol{T_L}$ is applied from 0 to -50 Nm in 0.15 seconds. Equation 10 described q-axis current approximation $\boldsymbol{i_{qs}}$.

$$\tilde{\iota}_{qs} = 2\left[i_{qs}^{re}cos(\omega_s t) - i_{qs}^{im}sin(\omega_s t)\right] \tag{10}$$

Where i_{qs}^{re} is q-axis current real part, and i_{qs}^{im} is the imaginary part.

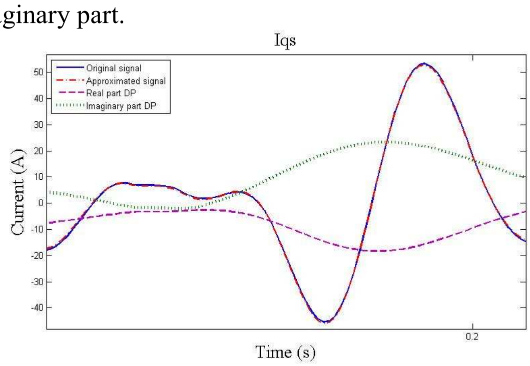


Fig. 7. Reconstruction of the signal.

D. Parameters variation analysis.

An interesting application of dynamic phasors is in controller design, so it is important to implement filters that compute Fourier coefficients in real time [16], as voltages and mechanical torque. In this work these periodic signals are computed off-line with equation 2 and their values are shown in table I.

TABLE I. Fourier coefficients of periodic signals.

$\cos(\omega_s t)$	$a_1 = a_{-1} = \frac{1}{2};\ a_k = 0, \forall \lvert k \rvert \neq 1$
$\sin(\omega_s t)$	$a_1 = a_{-1} = \frac{1}{2j};\ a_k = 0, \forall \lvert k \rvert \neq 1$
$x(t) = 1$	$a_0 = 1;\ a_k = 0, \forall k \neq 0$

Hence, when exists variation in voltages amplitude, frequency or mechanical load, their Fourier coefficients must be updated.

$$V_{ds} = V * sin\left(\omega_s t + \frac{\pi}{2}\right) \quad (11)$$

$$V_{qs} = V * sin(\omega_s t) \quad (12)$$

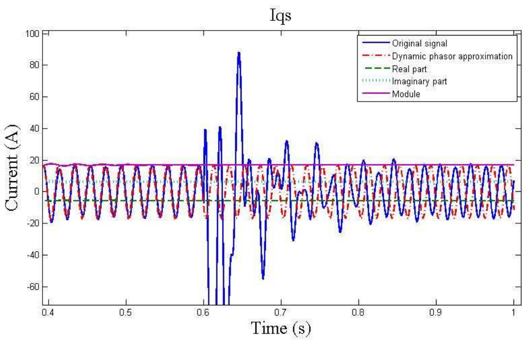
Thus, original dq-voltages define by equations 11 and 12 have their respective Fourier coefficients given in 13 and 14 that shall be applied to dynamic phasors model of model 8.

$$V_{dsFD} = \frac{1}{2}V \quad (13)$$

$$V_{qsFD} = -j\frac{1}{2}V \quad (14)$$

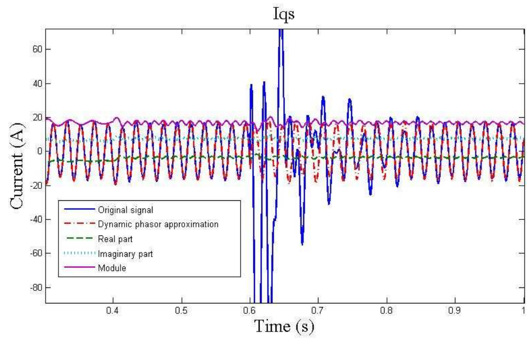
This update requirement is evident as show figures 9 and 10, where a change of parameters are made in next form: at 0.4 seconds, an increase and decrease of 2.5% in amplitude voltage are made in V_{ds} and V_{qs} respectively, and frequency is change from 50 to 50.5 Hz at 0.6 seconds. Mechanical load is defined at a constant value of -50 Nm.

In figure 9a can be appreciated how the approximation with dynamic phasors is a little distorted when there is a voltage variation at 0.4 seconds, and at 0.6 seconds it is completely lost when frequency is changed. However, in figure 9.b with variation parameters update, the approximation of the signal is good with voltage amplitude variation and when the frequency is changed, after 0.3 seconds the approximation is recovered.



a) Without update

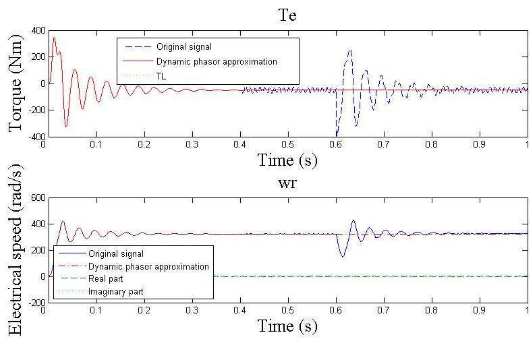
Similar results are observed in figure 10a, where electromagnetic torque dynamic phasors approximation does not follow original signal neither unbalanced voltages nor with the frequency variation, and the same is observed with electrical speed dynamic.



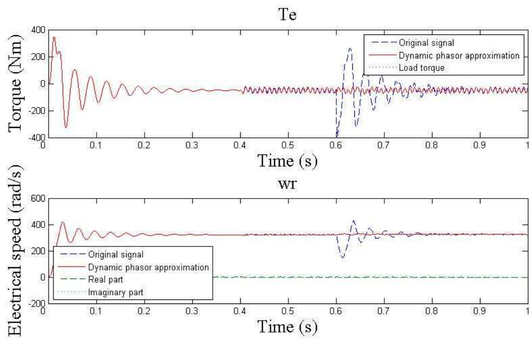
b) With update.

Fig. 9 q-axis current dynamic with electrical parameters variation.

When parameters are updated, the approximations of the signals are good as shown in figure 10b, where voltage and frequency variations are included in dynamic phasors model. Also can be observed, how there are oscillations in electromagnetic torque and electrical speed.



a) With update.



b) Without update.

Fig. 10. Electromagnetic torque and electrical speed dynamics with electrical parameters variation.

E. Active and reactive powers with dynamic phasors.

In DQ0 transformation, active and reactive powers are computed with equations 15 and 16 [17]

$$\mathbf{P} = \frac{3}{2}\left(\mathbf{V_d I_d} + \mathbf{V_q I_q} + \mathbf{2V_0 I_0}\right) \quad (15)$$

$$\mathbf{Q} = \frac{3}{2}\left(\mathbf{V_q I_d} - \mathbf{V_d I_q}\right) \quad (16)$$

Where equation 16 is valid only for a balanced and harmonic-free system. In equation 15, if the system is balanced then $\mathbf{I_0 = 0}$.

In DQ0 dynamic phasors modelling these powers are computed with equations 17 and 18, and are validated in figure 11.

$$\mathbf{P_{DP} = 3\left[V_{ds}\left(i_{ds}^{re} + i_{ds}^{im}\right) + V_{qs}\left(i_{qs}^{re} + i_{qs}^{im}\right)\right]} \quad (17)$$

$$\mathbf{Q_{DP} = 3\left[V_{ds}\left(i_{qs}^{re} + i_{qs}^{im}\right) - V_{qs}\left(i_{ds}^{re} + i_{ds}^{im}\right)\right]} \quad (18)$$

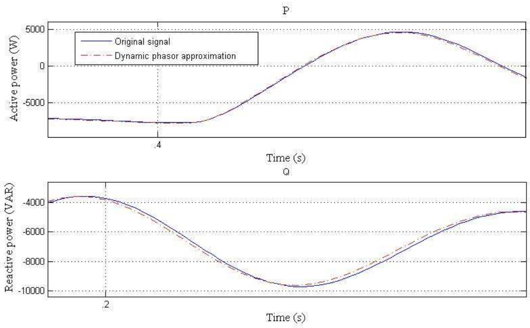


Fig.11. Comparison between traditional $\mathbf{PQ}$ and $\boldsymbol{PQ_{DP}}$ powers.

5. Conclusions

In this work a DQ dynamic phasors model of the wound rotor induction generator has been developed, whit four complex equations to describe stator and rotor currents dynamics, and two equation to describe the rotor electrical speed, one include dc component, and other include second harmonic. This model was employed to analyse dynamics under load, voltages amplitude and frequency variations. Simulation results in Matlab/Simulink showed the accurate and efficient of the proposed work. Active and reactive power dynamic phasors equations were presented.

Dynamic phasors approach offers a number of advantages over conventional methods: the selection and variation of Fourier coefficients *k,* gives a wider bandwidth in the frequency domain than traditional slow quasi-stationary models used in Transient Stability Programs and gives also the possibility of showing couplings between various quantities and addressing particular problems at different frequencies, however the number of differential equations increase.

As the variations of dynamic phasors are slower than the instantaneous quantities, they can be used to compute the fast electromagnetic transients with larger step sizes, so that it makes simulation potentially faster than conventional time domain like EMTP simulation. The dynamic phasors approach also allows an analytical insight into system sensitivities used to design controllers or protection schemes. It is our interest continuous researching in dynamic phasors applications to model hybrid (DC-AC, continuous-discrete) systems with power electronics like transfer and conditioning powers elements, and renewable energies technologies as WECS, PV and storage systems.

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